Elec 428 - Spring 2006
Homework 2

All homework assignments and projects in this course are covered by the Honor Code. You may work on the homework problems together but all written work must be exclusively your own (do not look at anyone else's written work or at solutions from previous years). Late homework assignments lose credit at the rate of 10% per day.

Due date: Friday, March 10

1. Suppose you were asked to generate random numbers for a quadratic distribution on the range (0,1); i.e., the density function is a function $f_X(x) = ax^2 + bx + c$ for $0 \le x \le 1$, and $f_X(x) = 0$ otherwise. You are also told that $f_X(0) = 0$ and $f_X(1) = 0$.

   (a) What is α if you decide to use an acceptance/rejection algorithm based on bounding $f_X(x)$ by a constant? How manyiterations on average will be required for each value accepted?

   (b) A symmetric triangular function $f_X(x)$ on (0,1) is a piecewise linear, continuous function with a slope of k between 0 and 0.5 and a slope of $-k$ between 0.5 and 1, for some $k > 0$. Also, $f_X(0) = f_X(1)$. Would there be any advantage in using a symmetric triangular function on the interval (0,1) as a bounding function, rather than a constant? Why or why not? Be specific.

2. A random number generator is supposed to generate uniformly distributed integers between 0 and 99. In one test, it produces the following sequence of 20 values:

   17,63,9,82,5,44,13,20,2,93,47,50,56,16,80,4,35,39,94,79

   Is it likely that the random number generator is a poor one? Why or why not? Be as specific as possible in explaining how you reached your conclusion.

3. A random number generator is supposed to produce independent random numbers from a discrete $U(1,20)$distribution (i.e., the values produced by the generator are supposed to be integers uniformly distributed between 1 and 20, inclusive, and the values are supposed to be independent of one another). When the random number generator is tested, it produces the following sequence of values:

   1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20

   Show that it is statistically unlikely that this sequence is random, because of correlation among the values.

4. Suppose you want to use a *truncated exponential distribution* over the range (0,1) in a stochastic simulation. A truncated exponential distribution over this range has the form $F_X(x) = k\left(1 - e^{-\lambda x}\right)$ for $0 \le x \le 1$ and $F_X(x) = 0$ otherwise. Describe two algorithms for generating random numbers from this distribution. Be precise. You may assume that you have a $U(0,1)$ random number generator to use. Compare your algorithms on the

basis of efficiency, i.e., on the amount of computation required to produce a single random number from a truncated exponential distribution.

5. The density function for the standard Cauchy distribution is

$$f_X(x) = \frac{1}{\pi(1+x^2)}, \quad -\infty \le x \le \infty$$

Describe a method, in detail, for generating values of a random variable with a standard Cauchy distribution.