Elec 428
Spring Semester 2006

First Exam

This is a 60-minute, open-book, open-notes, take-home exam. Work the exam on standard 8.5" x 11" paper, **one side only**. If a problem statement appears ambiguous or incomplete, make any *reasonable* assumptions that you feel are necessary to solve the problem and state those assumptions as part of your solution. Return the exam with your solution sheets. The exam is due at 11:59 PM on Friday, March 24. You may use a calculator or other calculating aid, but do not use matlab, preprogrammed algorithms, etc. (i.e., just use the basic arithmetic capabilities of your calculator/computer; square roots, squares, trigonometric functions, and other elementary math functions are okay, as are sample means and sum of squared values).

Show all your work. No partial credit for incorrect answers can be given otherwise, and full credit for correct answers may not be given without it.

SIGN THE PLEDGE

First Exam

1. Suppose you have $10,000 to spend on constructing a computer system intended to service 10 users. The system consists of 10 terminals (one per user), a single CPU, and a set of identical disk drives. The CPU comes in three models: A, B, and C. Each user job requires an average of 40 ms of service time per visit to the model A CPU, 20 ms of service time on model B, and 12 ms of service time on model C. Each user job requires an average of 40 ms of service time per visit to a disk. The disk load is balanced regardless of the number of disks. That is, if the system has a single disk drive, 100% of the user requests go to that disk; if the system has two disk drives, 50% of the user requests go to each disk; etc. User think times are 10 seconds on average.

   A user job leaving the CPU returns to its terminal with probability 0.01 or goes to a disk with probabiltiy 0.99. Jobs leaving a disk always go back to the CPU.

   A model A CPU costs $4,000, model B costs $6,000, and model C costs $7,000. Each disk drive costs $1,000.

   (a) What is the maximum possible steady-state system throughput at 10 users for each CPU model and as many disks as the CPU price and the total budget of $10,000 allows? (30 pts)

   (b) For each system in (a), what is the system throughput that falls exactly halfway between the minimum and maximum bounds for throughput? (5 pts)

2. (a) Use a permutation test with a subsequence length of 3 to analyze the following sequence of 100 outputs from what is supposed to be a discrete U(0,99) random number generator. What is your conclusion about the quality of that generator? (30 pts)

   27,18,28,18,28,45,90,45,23,53,60,28,74,71,35,26,62,49,77,57

   24,70,93,69,99,59,57,49,66,96,76,27,72,40,76,63,03,53,54,75

   94,57,13,82,17,85,25,16,64,27,42,74,66,39,19,32,00,30,59,92

   18,17,41,35,96,62,90,43,57,29,00,33,42,95,26,05,95,63,07,38

   13,23,28,62,79,43,49,07,63,23,38,29,88,07,53,19,52,51,01,90

   (b) How could you use a permutation test to test the random number generator if you had only the first 50 outputs instead of all 100? (5 pts)

3. (a) Find a 95% confidence interval for the true mean of the first 20 numbers in the sequence of numbers in problem 2, assuming that the numbers in problem 2 are the values of 100 independent and identically distributed random variables. Give the endpoints, not just the width of the confidence interval. (25 pts)

   (b) Suppose you only wanted to be 50% sure that your confidence interval encompassed the true mean of the distribution of the random variables. What would the confidence interval be? (5 pts)