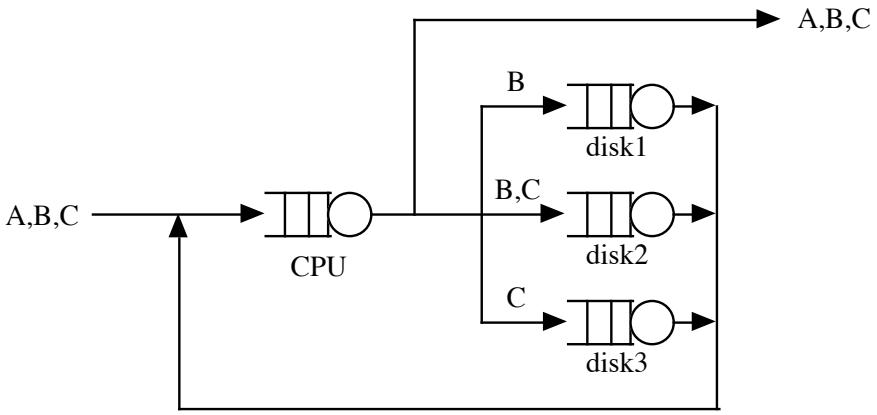
1. A computer system consisting of a CPU and three disks processes three types of jobs A, B, and C. The open queueing network model for the system has a product-form solution.



The model inputs are:

$\lambda_A = 0.01$	$\lambda_B = 0.05$	$\lambda_C = 0.04$
$D_{A,CPU} = 10$	$D_{B,CPU} = 5$	$D_{C,CPU} = 8$
$D_{A,disk1} = 0$	$D_{B,disk1} = 15$	$D_{C,disk1} = 0$
$D_{A,disk2} = 0$	$D_{B,disk2} = 12$	$D_{C,disk2} = 6$
$D_{A,disk3} = 0$	$D_{B,disk3} = 0$	$D_{C,disk3} = 12.5$

(a) Is this system stable? Prove your answer.

First compute the per-class utilization of each of the queues.

$$\rho_{A,cpu} = \lambda_{A,cpu} D_{A,cpu} = 0.01(10) = 0.1$$
$$\rho_{A,disk1} = \rho_{A,disk2} = \rho_{A,disk3} = 0$$

$$\rho_{B,cpu} = \lambda_{B,cpu} D_{B,cpu} = 0.05(5) = 0.25$$
$$\rho_{B,disk1} = \lambda_{B,disk1} D_{B,disk1} = 0.05(15) = 0.75$$
$$\rho_{B,disk2} = \lambda_{B,disk2} D_{B,disk2} = 0.05(12) = 0.6$$
$$\rho_{B,disk3} = 0$$

$$\rho_{C,cpu} = \lambda_C D_{C,cpu} = 0.04(8) = 0.32$$
$$\rho_{C,disk1} = 0$$
$$\rho_{C,disk2} = \lambda_C D_{C,disk2} = 0.04(6) = 0.24$$
$$\rho_{C,disk3} = \lambda_C D_{C,disk3} = 0.04(12.5) = 0.5$$

Total queue utilization is the sum of the per-class utilizations:

$$\rho_{cpu} = \rho_{A,cpu} + \rho_{B,cpu} + \rho_{C,cpu} = 0.1 + 0.25 + 0.32 = 0.67 < 1$$
$$\rho_{disk1} = \rho_{A,disk1} + \rho_{B,disk1} + \rho_{C,disk1} = 0 + 0.75 + 0 = 0.75 < 1$$
$$\rho_{disk2} = \rho_{A,disk2} + \rho_{B,disk2} + \rho_{C,disk2} = 0 + 0.6 + 0.24 = 0.84 < 1$$
$$\rho_{disk3} = \rho_{A,disk3} + \rho_{B,disk3} + \rho_{C,disk3} = 0 + 0 + 0.5 = 0.5 < 1$$

$\therefore$ The system is stable.

(b) What are the average system response times for job classes A, B, and C? How many customers total (of all classes) are in the system on the average?

Average system response times for each job class at each queue:

$$R_{A,cpu} = \frac{D_{A,cpu}}{1-\rho_{cpu}} = \frac{10}{1-0.67} = \frac{10}{0.33} \approx 30$$
$$R_{B,cpu} = \frac{D_{B,cpu}}{1-\rho_{cpu}} = \frac{5}{0.33} \approx 15$$
$$R_{C,cpu} = \frac{D_{C,cpu}}{1-\rho_{cpu}} = \frac{8}{0.33} \approx 24$$

$$R_{A,disk1} = 0$$
$$R_{B,disk1} = \frac{D_{B,disk1}}{1-\rho_{disk1}} = \frac{15}{0.25} = 60$$
$$R_{C,disk1} = 0$$

$$R_{A,disk2} = 0$$
$$R_{B,disk2} = \frac{D_{B,disk2}}{1-\rho_{disk2}} = \frac{12}{0.16} = 75$$
$$R_{C,disk2} = \frac{D_{B,disk2}}{1-\rho_{disk2}} = \frac{6}{0.16} = 37.5$$

$$R_{A,disk3} = 0$$
$$R_{B,disk3} = 0$$
$$R_{C,disk3} = \frac{D_{C,disk3}}{1-\rho_{disk3}} = \frac{12.5}{0.5} = 25$$

Average system response time for each job class:

$$
\begin{aligned}
R_A &= R_{A,cpu} + R_{A,disk1} + R_{A,disk2} + R_{A,disk3} \\
&= 30 + 0 + 0 + 0 = 30 \\
R_B &= R_{B,cpu} + R_{B,disk1} + R_{B,disk2} + R_{B,disk3} \\
&= 15 + 60 + 75 + 0 = 150 \\
R_C &= R_{C,cpu} + R_{C,disk1} + R_{C,disk2} + R_{C,disk3} \\
&= 24 + 0 + 37.5 + 25 = 86.6
\end{aligned}
$$

Average number of jobs of all classes in the system:

$$
\begin{aligned}
N &= N_A + N_B + N_C = \lambda_A R_A + \lambda_B R_B + \lambda_C R_C \\
&= 0.01(30) + 0.05(150) + 0.04(86.5) \\
&= 0.3 + 7.5 + 3.46 = 11.26
\end{aligned}
$$