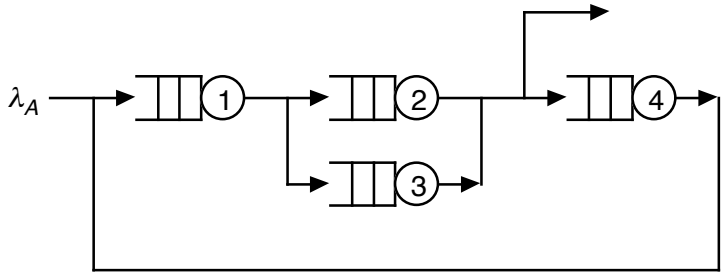
1. The queueing network model in the figure below has three job classes A, B, and C. A is an open class, while B and C are closed classes. The class A arrival rate is $\lambda_A = 1$. Class B has two jobs, while class C has one job.



Class A jobs leaving queue 1 go to queue 2 with probability 0.2 or to queue 3 with probability 0.8. Class A jobs leaving queues 2 and 3 always leave the system.

Class B jobs departing queue 1 are routed to queue 2 or queue 3 with equal probability. Class C jobs departing queue 1 always go to queue 2.

The queueing network is product-form. The mean service times for the job classes at the queues are:

$$\frac{1}{\mu_{A,1}} = 0.2 \quad \frac{1}{\mu_{B,1}} = 0.05 \quad \frac{1}{\mu_{C,1}} = 0.1$$
$$\frac{1}{\mu_{A,2}} = 1 \quad \frac{1}{\mu_{B,2}} = 0.1 \quad \frac{1}{\mu_{C,2}} = 0.4$$
$$\frac{1}{\mu_{A,3}} = 0.5 \quad \frac{1}{\mu_{B,3}} = 0.04$$
$$\frac{1}{\mu_{B,4}} = 0.04 \quad \frac{1}{\mu_{C,4}} = 0.2$$

Find the average response time for each job class, and find the utilization of each queue by all job classes combined.

Find the queue utilizations by class A jobs.

$$\rho_{A,1} = \frac{\lambda_A}{\mu_{A,1}} = 1(0.2) = 0.2$$

$$\rho_{A,2} = \frac{\lambda_A}{\mu_{A,2}} = 0.2(1)(1) = 0.2$$

$$\rho_{A,3} = \frac{\lambda_A}{\mu_{A,3}} = 0.8(1)(0.5) = 0.4$$

Now inflate the demands for class B and C jobs at each of the queues. First, compute the visit ratios for class B jobs.

$$\underline{\underline{P}} = \begin{bmatrix} 0 & 0.5 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad \Rightarrow \quad \begin{aligned} V_4 &= V_1 \\ 0.5V_1 &= V_2 \\ 0.5V_1 &= V_3 \end{aligned} \quad \Rightarrow \quad \underline{V} = (1, 0.5, 0.5, 1)$$

(setting $V_1 = 1$). Next compute the (uninflated) demands.

$$D_{B,1} = 1(0.05) = 0.05$$
$$D_{B,2} = 0.5(0.1) = 0.05$$
$$D_{B,3} = 0.5(0.04) = 0.02$$
$$D_{B,4} = 1(0.04) = 0.04$$

Now inflate the class B demands to account for the class A utilizations.

$$D^*_{B,1} = \frac{D_{B,1}}{1-\rho_{A,1}} = \frac{0.05}{1-0.2} = \frac{1}{16}$$
$$D^*_{B,2} = \frac{D_{B,2}}{1-\rho_{A,2}} = \frac{0.05}{1-0.2} = \frac{1}{16}$$
$$D^*_{B,3} = \frac{D_{B,3}}{1-\rho_{A,3}} = \frac{0.02}{1-0.4} = \frac{1}{30}$$
$$D^*_{B,4} = \frac{D_{B,4}}{1-\rho_{A,4}} = \frac{0.04}{1} = \frac{1}{25}$$

For class C, obviously $V_1 = V_2 = V_4 = 1$ and $V_3 = 0$.

$$D_{C,1} = 0.1 \quad D^*_{C,1} = \frac{0.1}{0.8} = \frac{1}{8}$$
$$D_{C,2} = 0.1 \quad D^*_{C,2} = \frac{0.4}{0.8} = \frac{1}{2}$$
$$D_{C,4} = 0.1 \quad D^*_{C,4} = \frac{0.2}{1} = \frac{1}{5}$$

Use MVA for the closed classes with inflated demands.

	0B,0C	1B,0C	0B,1C	1B,1C	2B,0C	2B,1C
$R_{B,1}$	-	0.0625	-	0.0720	0.0822	0.0801
$R_{B,2}$	-	0.0625	-	0.1004	0.0822	0.0871
$R_{B,3}$	-	0.0333	-	0.0333	0.0389	0.0377
$R_{B,4}$	-	0.0400	-	0.0497	0.0481	0.0478
$R_{C,1}$	-	-	0.125	0.1644	-	0.2067
$R_{C,2}$	-	-	0.500	0.6576	-	0.8270
$R_{C,4}$	-	-	0.200	0.2403	-	0.2765
X_B	-	5.0420	-	3.9154	7.9554	7.9145
X_C	-	-	1.2121	0.9414	-	0.7632
$Q_{B,1}$	0	0.3151	0	0.2819	0.6539	0.6339
$Q_{B,2}$	0	0.3151	0	0.3031	0.6539	0.6893
$Q_{B,3}$	0	0.1681	0	0.1305	0.3094	0.2984
$Q_{B,4}$	0	0.2017	0	0.1946	0.3826	0.3783
$Q_{C,1}$	0	0	0.1515	0.1548	0	0.1578
$Q_{C,2}$	0	0	0.6061	0.6191	0	0.6312
$Q_{C,4}$	0	0	0.2424	0.2262	0	0.2110

Find the response times for the open class A.

$$\begin{array}{lcll} D_{A,1} = \frac{1}{\mu_{A,1}} = 0.2 & & D^*_{A,1} = \frac{0.2}{1-0.2} = \frac{1}{4} & R_{A,1} = \frac{1}{4}(1.7917) = 0.4479 \\ D_{A,2} = \frac{1}{\mu_{A,2}} = 1 & \Rightarrow & D^*_{A,2} = \frac{1}{1-0.2} = \frac{5}{4} & \Rightarrow \quad R_{A,2} = \frac{5}{4}(2.3205) = 2.9006 \\ D_{A,3} = \frac{1}{\mu_{A,3}} = 0.5 & & D^*_{A,3} = \frac{0.5}{1-0.4} = \frac{5}{6} & R_{A,3} = \frac{5}{6}(1.2984) = 1.0820 \end{array}$$

Total queue utilizations:

$$\rho_1 = \rho_{A,1} + \rho_{B,1} + \rho_{C,1} = 0.2 + 7.9145(.05) + 0.7632(0.1) = 0.6720$$
$$\rho_2 = \rho_{A,2} + \rho_{B,2} + \rho_{C,2} = 0.2 + 7.9145(.05) + 0.7632(0.4) = 0.9010$$
$$\rho_3 = \rho_{A,3} + \rho_{B,3} + \rho_{C,3} = 0.4 + 7.9145(.02) + 0.7632(0) = 0.5583$$
$$\rho_4 = \rho_{A,4} + \rho_{B,4} + \rho_{C,4} = 0 + 7.9145(.04) + 0.7632(0.2) = 0.4692$$