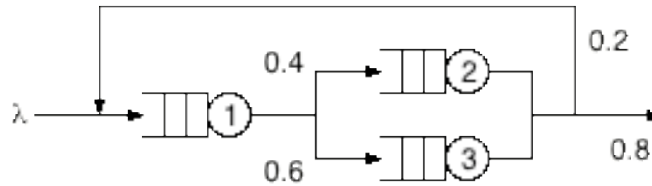


1. The following figure shows a single class queueing network of three queues that has a product form solution.



Jobs arrive from the outside with rate $\lambda = 1/200$. Jobs leaving queue 1 go to queue 2 with probability 0.4 or to queue 2 with probability 0.6. Jobs leaving queue 2 or queue 3 leave the system with probability 0.8 or go to queue 1 with probability 0.2. The per visit demands for queues 1, 2, and 3 are 50, 100, and 200, respectively.

- (a) What is the steady state probability that queues 2 and 3 both have 0 jobs or both have 1 job or both have 2 jobs?

Since this is an open product form queueing network, the joint queue length probability distribution is of the form

$$\Pr[n_1, n_2, n_3] = \rho_1^{n_1} (1 - \rho_1) \rho_2^{n_2} (1 - \rho_2) \rho_3^{n_3} (1 - \rho_3)$$

where n_i and ρ_i are, respectively, the number of jobs in and the utilization of queue i . To obtain the utilizations, find the queue arrival rates.

$$\lambda_2 = 0.4\lambda_1$$

$$\lambda_3 = 0.6\lambda_1$$

$$\lambda_1 = \lambda + 0.2\lambda_1 \Rightarrow \lambda_1 = \lambda/0.8 = 1.25\lambda$$

$$\lambda_2 = 0.5\lambda$$

$$\lambda_3 = 0.75\lambda$$

Since $\lambda = 1/200$, the utilizations are

$$\rho_1 = \lambda_1 \cdot 50 = 0.3125$$

$$\rho_2 = \lambda_2 \cdot 100 = 0.25$$

$$\rho_3 = \lambda_3 \cdot 200 = 0.75$$

Note that all of the queues have utilizations less than one; the network has a steady state solution.

The probability that queues 2 and 3 both have 0 or 1 or 2 jobs is

$$\begin{aligned}
 & \sum_{n_1=0}^{\infty} \{ \Pr[n_1, 0, 0] + \Pr[n_1, 1, 1] + \Pr[n_1, 2, 2] \} \\
 &= \sum_{n_1=0}^{\infty} \rho_1^{n_1} (1 - \rho_1) \{ \rho_2^0 (1 - \rho_2) \rho_3^0 (1 - \rho_3) + \rho_2^1 (1 - \rho_2) \rho_3^1 (1 - \rho_3) + \rho_2^2 (1 - \rho_2) \rho_3^2 (1 - \rho_3) \} \\
 &= (1 - \rho_1) (1 - \rho_2) (1 - \rho_3) \sum_{n_1=0}^{\infty} \rho_1^{n_1} \{ \rho_2^0 \rho_3^0 + \rho_2^1 \rho_3^1 + \rho_2^2 \rho_3^2 \} \\
 &= (.6875)(.75)(.25) \sum_{n_1=0}^{\infty} \rho_1^{n_1} \{ 1 + (.25)(.75) + (.25)^2 (.75)^2 \} \\
 &= (.1289)(1.2227) \frac{1}{1 - \rho_1} = \frac{.1576}{.6875} = .2292
 \end{aligned}$$

- (b) If the arrival rate is from the outside is doubled, what is the steady state probability that queues 2 and 3 both have 0 jobs or both have 1 job or both have 2 jobs?

The system does not have a steady state, since the utilization of queue 3 would be greater than 1.