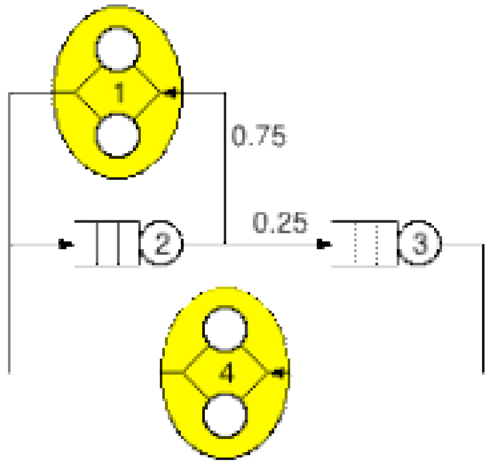
1. In the following closed product-form queueing network, queues 1 and 4 are infinite server (IS) queues while queues 2 and 3 are single-server queues.



The service rates are as follows ($\mu_1(1)$ and $\mu_4(1)$ are the service rates for queues 1 and 4, respectively, when there is one job in the queue):

$\mu_1(1) = 20$ $\mu_2 = 5$ $\mu_3 = 2$ $\mu_4(1) = 10$

If there are three jobs in the system, what are the average numbers of jobs at queues 1 and 4?

Any queue will work, but assume that we choose queue 2 as the reference queue ($V_2 = 1$). It should be obvious that $V_1 = 0.75$, $V_3 = 0.25$, and $V_4 = 0.25$. With these visit ratios and the above service rates, compute the total service demands $D_i = V_i \cdot (1/\mu_i)$:

$D_1 = 0.0.0375$ $D_2 = 0.2$ $D_3 = 0.125$ $D_2 = 0.025$

Applying MVA:

	$N = 0$	$N = 1$	$N = 2$	$N = 3$
R_2	-	0.2000	0.3032	0.4284
R_3	-	0.1250	0.1653	0.2028
X	-	2.5806	3.7665	4.3246
Q_2	0	0.5161	1.1420	1.8527
Q_3	0	0.3226	0.6226	0.8770

With the throughput for 3 jobs, we can compute the expected numbers of jobs in queues 1 and 4:

$$Q_1 = XD_1 = 4.3246(0.0375) = 0.1622$$

$$Q_4 = XD_4 = 4.3246(0.025) = 0.1081$$