1. A computer systems performance consultant is asked to study a system consisting of a CPU and two disks A and B. She specifically is asked to investigate the benefits of replacing disk A with a disk twice as fast (the average total service time is half that of the original disk).

   The consultant studies the system (with the original disk A) over a 100,000-second period. She finds that during that time 10,000 jobs arrive at the system and approximately 10,000 jobs complete. The CPU, disk A, and disk B are busy for 50,000, 80,000, and 40,000 seconds, respectively.

   The consultant, who had just finished taking ELEC 428, decides for reasons that may or may not be obvious that the system can be modeled as a product-form queueing network, and that all jobs behave similarly (at least in a stochastic sense).

   How much will the system response time be reduced by replacing disk A by a disk that is twice as fast? How many disk access requests were at disk A on the average, and how many would be at disk A's replacement under the same workload?

Having made the decision to use a product-form queueing network model of the system, the consultant needs to come up with estimates for two sets of numbers: the CPU and disk utilizations and their total demands. These estimates are:

$$\rho_{cpu} = \frac{50{,}000}{100{,}000} = 0.5 \quad D_{cpu} = \frac{50{,}000}{10{,}000} = 5$$
$$\rho_A = \frac{80{,}000}{100{,}000} = 0.8 \quad D_A = \frac{80{,}000}{10{,}000} = 8$$
$$\rho_B = \frac{40{,}000}{100{,}000} = 0.4 \quad D_B = \frac{40{,}000}{10{,}000} = 4$$

The average system response time is

$$R = \frac{D_{cpu}}{1-\rho_{cpu}} + \frac{D_A}{1-\rho_A} + \frac{D_B}{1-\rho_B} = \frac{5}{0.5} + \frac{8}{0.2} + \frac{4}{0.6} = \frac{170}{3}$$

With disk A replaced by a disk that is twice as fast, both the utilization of and total demand for disk A will be reduced: $\rho_A' = 0.4$ and $D_A' = 4$. The system response time with the new disk will be

$$R' = \frac{5}{0.5} + \frac{4}{0.6} + \frac{4}{0.6} = \frac{70}{3}$$

Hence the system response time will be reduced by 100/3 sec.

The arrival rate for jobs is $10,000/100,000 = 0.1$ jobs per second. With the original disk, the average number of jobs at disk A is

$$Q_A = \lambda R_A = \lambda D_A / (1 - \rho_A) = 0.1 \cdot 40 = 4$$

Replacing A with the faster disk gives

$$Q_A' = \lambda R_A' = \lambda D_A' / (1 - \rho_A') = 0.1 \cdot (20/3) = 2/3$$