

Math 101, Spring 2008, Exam 1

The exam consists of seven questions. You must show all your work to receive full credit. Please indicate your final answer clearly. Write the honor pledge on your exam when finished. Good luck!

1) Find the derivative with respect to x .

a) $f(x) = (x^{10} + 7x^4 - 3x + 3)(\csc x)$

b) $y = x^{\tan x^2}$

c) $h(x) = e^{x^2 \sin x}$

2) Find the equation of the line tangent to the curve $g(x) = \left(\frac{x+1}{x-1}\right)^3$ at the point $(0, -1)$.

3) Find and describe the discontinuities of the function $f(x) = \frac{x^2+5x-14}{x^2-3x+2}$.

4) Use the limit definition of derivative to find the derivative of $g(x) = \frac{x}{x+3}$.

5) I want a new soup pot. Create-A-Pot, Inc. charges $\$5/\text{in}^2$ for copper, $\$2/\text{in}^2$ for stainless steel, and $\$1/\text{in}^2$ for glass. I want a one gallon cylindrical pot with a copper bottom, a stainless steel side, and a glass top. The radius of the pot should be between 1 and 10 inches. Keeping in mind the fact that there are approximately 81π cubic inches in a gallon, what is the radius of the pot that minimizes my cost?

6) If $\lim_{x \rightarrow 0^+} f(x) = A$ and $\lim_{x \rightarrow 0^-} f(x) = B$, find the limit.

a) $\lim_{x \rightarrow 0^+} f(x^3 - x^2)$

b) $\lim_{x \rightarrow 0^-} f(x^3 - x^2)$

7) Use the intermediate value theorem to show that the function $f(x) = 2 + 3 \cos x$ has a root on the interval $[0, \pi]$. What properties of $f(x)$ allow you to use the intermediate value theorem?

Bonus: Draw me a picture.