

Math 101, Spring 2008, Practice Exam 2

- 1) a) State the first part of the fundamental theorem of calculus.  
b) Use the fundamental theorem of calculus to find the derivative of the function

$$g(x) = \int_{\frac{\pi}{4}}^{x^4} \frac{\cos(\sqrt{t})dt}{t^{3/2} + 7t}$$

- c) Use the fundamental theorem of calculus to find the derivative of the function

$$h(x) = \int_5^x (t^{5000} e^{7t^3}) dt$$

- 2) Find the following limits:

- a)  $\lim_{x \rightarrow \infty} (\sqrt{x-7} - \sqrt{x})$   
b)  $\lim_{x \rightarrow 0} \frac{e^x - 1}{e^x - e^{-x}}$   
c)  $\lim_{x \rightarrow 1} \frac{7x}{x-1} - \frac{1}{x^2-1}$   
d)  $\lim_{x \rightarrow 0} x^{\sin x}$   
e)  $\lim_{x \rightarrow \infty} \frac{5x^4 - 3x^2 + 7}{-3x^4 + 7x - 2}$

- 3) Show that the function  $f(x) = x^3$  satisfies the hypotheses of the mean value theorem on the interval  $[0, 4]$ . Find all numbers  $c$  in that interval that satisfy the conclusion of the mean value theorem.

- 4) Find the definite integral:

- a)  $\int_0^{\frac{\pi}{2}} \sec^2 \frac{x}{2} dx$   
b)  $\int_1^4 \frac{dx}{x^2}$   
c)  $\int_2^4 \frac{4dx}{x}$   
d)  $\int_0^{\ln 4} 3e^{\frac{x}{2}} dx$

- 5) Consider the function  $f(x) = x^2 + x + 1$  on the interval  $I = [0, 5]$ .

a) Using a partition of  $I$  into 5 regular subintervals, use a right-endpoint sum to compute the Riemann sum for  $f(x)$ .

b) Without completing part c, is this an overestimate or an underestimate of the integral?

- c) Using the Riemann sum definition of the integral, find  $\int_0^5 f(x) dx$ .

- 6) a) Find  $\frac{dy}{dx}$  when  $y$  is defined implicitly by the equation  $2y = x^2 + \sin y$ .  
 b) Find the first two derivatives of  $y$  with respect to  $x$  when  $y$  is defined implicitly by the equation  $2x^3 - 3y^2 = 7$ .

- 7) Let  $f(x) = x^4 + 2x^3$ .  
 a) Find and classify all critical points of the graph of  $y = f(x)$ .  
 b) Where is  $f(x)$  increasing and decreasing?  
 c) Sketch a graph of  $f(x)$ . Label local extrema and inflection points. Be sure to include behavior "at infinity" and any asymptotes.

8)

$$f(x) = \frac{x^2 - 3}{2x - 4}$$

$$f'(x) = \frac{2(x - 3)(x - 1)}{(2x - 4)^2}$$

$$f''(x) = \frac{8}{(2x - 4)^3}$$

Use this information to sketch a graph of  $x$ . Label local extrema and inflection points. Be sure to include behavior "at infinity" and any asymptotes.

- 9) A 13-foot ladder is leaning against a house when its base starts to slide away. By the time the base is 12 ft from the house, the base is moving at the rate of 5 ft/sec.

- a) How fast is the top of the ladder sliding down the wall then?  
 b) At what rate is the area of the triangle formed by the ladder, wall, and ground changing then?  
 c) At what rate is the angle  $\theta$  between the ladder and the ground changing then?

- 10) Find the indefinite integral:

a)  $\int (x^3 - 3x + 7) dx$

b)  $\int \csc^2(\pi x) dx$

c)  $\int \frac{1}{3x} dx$

d)  $\int 5e^{-\frac{x}{2}} dx$

e)  $\int (3 \sin 2x - 2 \cos 3x) dx$

f)  $\int \frac{1}{(2x + 1)^2} dx$