

Math 101 Spring 2008 Practice Test 1

- 1) Find the derivative with respect to x .
 - a) $y = x^{(x^2)}$
 - b) $f(x) = (3x^2 + 7x - 10)(\csc(x^2))$
 - c) $g(x) = \sin(\sqrt[3]{x})$
 - d) $y = x \ln\left(\frac{x+1}{x-1}\right)$
 - e) $v(x) = \left(x - \frac{1}{x}\right)^7$
 - f) $f(x) = \sin e^x \cos e^x$
- 2) Use the limit definition of derivative to find the derivative.
 - a) $y = \frac{1}{\sqrt{x}}$
 - b) $x^2 - x + 2$
- 3) Evaluate the limit if it exists.
 - a) $\lim_{t \rightarrow 3} \frac{t^2 - 9}{t^2 - 5t + 6}$
 - b) $\lim_{x \rightarrow 0} x \cot 3x$
- 4) Use the intermediate value theorem to show that the function $y = 3x^3 + 3x^2 - 3x - 1$ has 3 roots in the interval $[-2, 2]$.
- 5) State the intermediate value theorem and use it to show that every real number has a cube root.
- 6) If $\lim_{x \rightarrow 0^+} f(x) = A$ and $\lim_{x \rightarrow 0^-} f(x) = B$, find the limit.
 - a) $\lim_{x \rightarrow 0^+} f(x^3 - x)$
 - b) $\lim_{x \rightarrow 0^-} f(x^3 - x)$
 - c) $\lim_{x \rightarrow 0^+} f(x^2 - x^4)$
 - d) $\lim_{x \rightarrow 0^-} f(x^2 - x^4)$
- 7) You are designing a **rectangular** poster containing a 50 in^2 **rectangle** of printing with margins of 4 inches each at top and bottom and 2 inches at each side. What overall dimensions will minimize the amount of paper used? **You may assume that the poster's width is between 5 and 54 inches.**
- 8) Find and describe the discontinuities of the function $f(x) = \frac{x^2 - 9}{x^2 - 5x + 6}$.
- 9) Find the equation of the line tangent to the curve $y = \sqrt{x^2 + e^x} - e$ when $x=1$.
- 10) Below is a table of values of two functions and their derivatives at various points. Let $h(x) = (f \circ g)(x)$ and $k(x) = (g \circ f)(x)$.

x	f(x)	f'(x)	g(x)	g'(x)
-3	-2	0	1	-2
-2	-1	1	3	2
-1	0	3	0	-1
0	1	3	0	1
1	2	2	1	-1
2	3	-1	-2	3
3	-3	-2	-2	0

- a) Does the graph of $h(x)$ have a horizontal tangent line **at any of the x values listed in the table**? If so, find the equation of that line (or lines).
- b) Does the graph of $k(x)$ have a horizontal tangent line **at any of the x values listed in the table**? If so, find the equation of that line (or lines).
- c) Is $h(x)$ increasing, decreasing, or neither at $x = 2$?
- 11) Let $f(x) = x^2 + 3x + 4$ if $x \geq 1$ and $f(x) = x^3 + 3x + 4$ if $x < 1$.
- a) Is $f(x)$ continuous for all x ?
- b) Is $f(x)$ differentiable for all x ?