Math 101 Spring 2008 Review for the final

1) Find the limit if it exists.

a) $\lim_{x\to 2} \frac{x^2+x-6}{x^2-4}$
b) $\lim_{x\to 0} \frac{\sin 5x}{x}$
c) $\lim_{x\to 0} \frac{1-\cos 3x}{2x^2}$
d) $\lim_{x\to 0^-} \frac{|5x|}{x}$
e) $\lim_{x\to 0} x^3 \cos \frac{1}{x}$
f) $\lim_{x\to 0} \sin\left(\frac{\pi}{2}\left(\frac{\sin x}{x}\right)\right)$
g) $\lim_{x\to 0} \frac{e^{\sin x}-1}{x}$
h) $\lim_{x\to\infty}(\ln 2x - \ln(x+1))$
i) $\lim_{x\to\infty} x^{1/\ln x}$

2) Use the epsilon-delta definition of the limit to prove that $\lim_{x\to 7} 2x-3 = 11$.

3) Find and describe any discontinuities of the function.

a) $f(x) = \frac{|5x|}{x}$
b) $f(x) = \frac{1}{x-2} - 3x$
c) $f(x) = \frac{x^2+3x-10}{x-2}$

4) Use the intermediate value theorem to show that there exists a number that is exactly one less than its cube.

5) a) Use the limit definition of the derivative to find the derivative of the function $g(x) = 4 - x^2$.

b) Find the equation of the tangent line to the curve when $x = 2$.

6) Find the derivative of the function with respect to x.

a) $F(x) = \int_{\sin x}^{7} t dt$
b) $y = \sqrt{x(x+1)}$
c) $g(x) = \frac{x^5-3x^3+\pi x-7}{x-2}$
d) $y = (\sin x)^{\cos x}$
e) $f(x) = e^{x^2} \sin x$

7) Below is the graph of the curve $y = \sqrt{\cos x}$ on one interval where it is defined.

a) Determine the interval shown.

b) Find the volume obtained by revolving this segment of the curve about the x-axis.

8) Below is the graph of the curve $y = \sin x$ from $x = 0$ to $x = \pi$. Set up, but do not solve, the following integrals.

a) The volume obtained by revolving the region bounded by that curve and the x-axis about the x-axis.

b) The volume obtained by revolving the region bounded by that curve and the x-axis about the y-axis.

c) The length of the curve.

d) The surface area obtained by revolving the curve about the x-axis.

e) The surface area obtained by revolving the curve about the y-axis.

9) Consider the region in the first quadrant of the plane bounded by the curves $x = y$ and $x = y^3$.

a) Find the area of the region.

b) Find the volume of the solid obtained by revolving this region about the x-axis.

c) Find the volume of the solid obtained by revolving this region about the y-axis.

10) Here is some information about a function $f(x)$.

x	f(x)	f'(x)	f"(x)
-1	15	-4	36
0	10	0	0
1	7	-8	-24
2	6	-16	0
3	-17	0	36

Assuming $f(x)$ and its first two derivatives are continuous and there are no other zeroes in the first two derivatives, sketch a graph of $f(x)$. Label any local extrema and inflection points.

11) Sketch a graph of the function $y = \frac{x^2-1}{x}$

12) A hot air balloon rising straight up from a level field is tracked by a range finder 500 ft from the lift-off point. At the moment the range finder's elevation angle is $\pi/4$, the angle is increasing at the rate of 0.14 rad/min. How fast is the balloon rising at that moment?

13) Find two positive numbers whose sum is 20 and whose product is as large as possible.

14) A police officer saw my car going 57 mph in Italy, TX at 4 pm. Two hours later, another officer saw my car going 45 mph in Paris, TX, 150 miles away. The speed limit on the highway was 60 mph. What theorem from calculus did the second officer use to give me a ticket?

15) Find the indefinite integral.

a) $\int z^3(1+z^4)^3 dz$

b) $\int \frac{\sqrt{x}}{3} + \frac{3}{\sqrt{x}} dx$

c) $\int x^2 e^{3x^3-9x} - e^{3x^3-9x} dx$

16) The velocity of a particle is given by the equation $v(t) = t^2 - 5t + 6$. It travels from time $t = 0$ to time $t = 4$.

a) Find its net displacement.

b) Find its total displacement.

17) Find the definite integral.

a) $\int_9^4 \frac{1-\sqrt{x}}{\sqrt{x}} dx$

b) $\int_0^{\pi/3} 4 \sec y \tan y dy$

c) $\int_{-\pi/2}^{\pi/2} \left(2 + \tan \frac{t}{2}\right) \sec^2 \frac{t}{2} dt$

d) $\int_0^1 \frac{x^3}{\sqrt{x^4+9}} dx$

18) $xy = \sin x + \cos y$. Find the derivative of y with respect to x, assuming y is defined implicitly as a function of x.

19) If $\frac{dy}{dt} = 12t(3t^2-1)^3$ and $y(1) = 3$, find y as a function of t.

20) Use the Riemann sum definition of the integral to compute $\int_1^3 4x - 2dx$.

I have not included any questions on 6.7-6.9, which will be bonus on the test. If you want to practice those sections, do the 6.7-6.9 optional assignment.