Math 101, Spring 2008: Two more homework questions due April 11-Solutions

1) Evaluate the integral

$$\int \left(\sqrt{1+\sin^2(x-1)}\,\sin(x-1)\cos(x-1)\right)dx$$

using the following sequences of substitutions:

a) $u = x-1$, followed by $v = \sin u$, then by $w = 1+v^2$.

$u = x-1$, so $du = dx$. The integral becomes

$\int \left(\sqrt{1+\sin^2(u)}\,\sin(u)\cos(u)\right)du.$

Now let $v = \sin u$, so $dv = \cos u du$. The integral becomes

$\int \left(\sqrt{1+v}^2 \cdot v\right)dv.$

Now let $w = 1+v^2$, so $dw = 2vdv$. The integral becomes

$$\begin{aligned}
\int \frac{1}{2}w^{\frac{1}{2}} &= \frac{1}{2}\cdot\frac{2}{3}w^{\frac{3}{2}} + C \\
&= \frac{1}{3}\left(1+v^2\right)^{\frac{3}{2}} + C \\
&= \frac{1}{3}\left(1+\sin^2 u\right)^{\frac{3}{2}} + C \\
&= \frac{1}{3}\left(1+\sin^2(x-1)\right)^{\frac{3}{2}} + C
\end{aligned}$$

b) $u = \sin(x-1)$, followed by $v = 1+u^2$.

Very similar to the above. If you have questions, ask me.

c) $u = 1 + \sin^2(x-1)$.
Same old, same old.

2) Evaluate the integral

$$\int \frac{\sin\sqrt{\theta}d\theta}{\sqrt{\theta\cos^3\sqrt{\theta}}}$$

For this one, we could let $u = \sqrt{\theta}$ or $u = \cos\sqrt{\theta}$. I'm choosing the latter.
Let $u = \cos\sqrt{\theta}$, so $du = -\sin\sqrt{\theta} \cdot \frac{1}{2\sqrt{\theta}}d\theta$. So $\frac{\sin\sqrt{\theta}d\theta}{\sqrt{\theta}} = -2du$. So the integral becomes

$$\begin{aligned}
& -2\int \frac{du}{u^{\frac{3}{2}}} \\
= \quad & -2\int u^{\frac{-3}{2}} du \\
= \quad & -2(-2u^{\frac{-1}{2}}) + C \\
= \quad & 4u^{\frac{-1}{2}} + C \\
= \quad & \frac{4}{\sqrt{cos\theta}} + C
\end{aligned}$$