Challenge problem solution

The problem was to find a limit in the indeterminate form 1^∞ such that when evaluated, the limit is your birth month plus one.

I was born in September, so I am aiming for the limit 10. There are a lot of ways to do this, but here is the one I used:

$\lim_{x\to 0}(1+x)^{\frac{\ln 10}{x}}$. To show that this works, first notice that is does indeed have the right indeterminate form, 1^∞. Now let $L = \lim_{x\to 0}(1+x)^{\frac{\ln 10}{x}}$.

$$\begin{aligned}
\lim_{x\to 0} \ln L &= \ln \lim_{x\to 0}(1+x)^{\frac{\ln 10}{x}} \\
\lim_{x\to 0} \ln L &= \lim_{x\to 0} \frac{\ln 10}{x} \ln(1+x) \\
\lim_{x\to 0} \ln L &= \lim_{x\to 0} \frac{\ln 10 \cdot \ln(1+x)}{x} \\
\lim_{x\to 0} \ln L &= \lim_{x\to 0} \frac{\ln 10}{1+x} \\
\ln L &= \ln 10
\end{aligned}$$

(To get from the third line to the fourth line, I used L'Hôpital's rule.) Since $\ln L = \ln 10$, $L = e^{\ln 10} = 10$, which is exactly what I wanted. If you were not born in September, you can get your month plus one (call it n) out by substituting $\ln n$ for $\ln 10$ in the first line. There are, of course, many other solutions.

What I am trying to emphasize in this problem is that 1^∞ really is an indeterminate form, and the rates at which the base is approaching 1 and the exponent is growing (or "approaching infinity") really make a huge difference in what the limit is. Also, math isn't just about starting with a problem and working until you get the right answer. Sometimes you start with the answer and have to figure out what the problem was.