THE IS-LM MODEL

1. The Textbook IS-LM Model

- To get an idea of the way the IS-LM model works, we shall, for the moment, ignore the labor market (even though this was a centerpiece of Keynes' own analysis). We replace the assumption of fixed nominal wages by one of fixed nominal prices. In the short run, equilibrium in the demand and supply of goods and in the demand and supply of money jointly determine the two endogenous variables – real output and the nominal interest rate (= real interest rate in the presence of fixed prices).

- The demand for goods consists of a demand for consumption and a demand for investment goods. Again we shall ignore some of the detail of Keynes' analysis by not distinguishing between "investment goods" and "consumption goods" but merely talk about aggregate output as a homogeneous good. Equilibrium in the goods market requires that

$$y = c + i \tag{1}$$

where, y = the level of output, c = consumption and i = investment.

- Households are assumed to decide on their level of consumption and savings by allocating their current income on the basis of current interest rates:

$$c = f(r, y) \tag{2}$$

(this contrasts with the real business cycle and growth models where consumption depends on *wealth* rather than current *income*. This issue will be discussed in more detail later.)

- Keynes postulates as a "fundamental psychological law" that the elasticity of consumption with respect to income is less than 1 so that, as income increases, consumption increases less than proportionately:

$$\frac{y}{f}\frac{\partial f}{\partial y} < 1 \tag{3}$$

We shall also assume that substitution effects dominate on average so that increases in r increase savings and so reduce consumption.

- Having determined their level of savings, households then allocate those savings between money and interest paying assets on the grounds that:

  (i) higher income increases the demand for money for transaction purposes, as in the classical quantity theory of money

  (ii) higher nominal (*and real with* p *fixed*) interest rates reduce the demand for money.

  This leads to the money demand function:

$$\frac{M}{p} = L(r, y) \tag{4}$$

with $L_r < 0$ and $L_y > 0$.

- Investment is determined by firms and depends positively on the current level of output and negatively on the current nominal rate of interest. In addition, Keynes argued that the desire to invest was unstable and much governed by "whims" and "herd instincts". We can represent this by an exogenous shift parameter α in the investment function:

$$i = g(r, y, \alpha) \tag{5}$$

- The product market equilibrium condition leads to a relationship between y and r called the IS curve:

$$y = f(r, y) + g(r, y, \alpha) \tag{6}$$

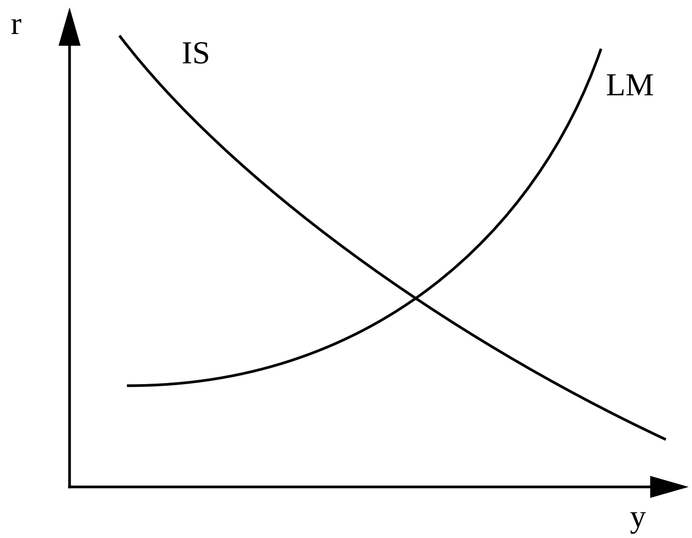
Totally differentiate this equilibrium condition to find

$$1 = \frac{\partial f}{\partial r}\frac{dr}{dy} + \frac{\partial f}{\partial y} + \frac{\partial g}{\partial r}\frac{dr}{dy} + \frac{\partial g}{\partial y} \tag{7}$$

which can be rearranged to yield

$$\frac{dr}{dy} = \frac{1 - \frac{\partial f}{\partial y} - \frac{\partial g}{\partial y}}{\frac{\partial f}{\partial r} + \frac{\partial g}{\partial r}} \tag{8}$$

- Then the assumptions on f and g imply the denominator of this derivative is negative. Also, it is usually assumed that the "marginal propensity to consume" out of income plus the marginal effect of higher output on investment are less than 1. In that case the IS curve will be negatively sloped in the (y, r) plane.



- Equilibrium in the supply and demand for money with M and p exogenously given also leads to a relationship between y and r called the LM curve:

$$L(r, y) = (M/p)^* \tag{9}$$

Totally differentiating this equilibrium condition (9) we find

$$\frac{\partial L}{\partial r}\frac{dr}{dy} + \frac{\partial L}{\partial y} = 0 \tag{10}$$

so that

$$\frac{dr}{dy} = -\frac{\partial L}{\partial y} \Big/ \frac{\partial L}{\partial r} \tag{11}$$

which is positively sloped in the (y, r) plane.

2. Comparative Statics

- We can use the IS-LM diagram to study the effects of changes in some of the exogenous variables. Consider first an increase in the money supply M. This will shift the LM curve to the right and leave the IS curve unaffected. Output y will increase, so there will be pro-cyclical movements in M which corresponds to our observations. However, r moves counter-cyclically and there is considerable doubt whether this corresponds with the evidence. Our observation of the data suggested r tended to be relatively high as the economy went into a downturn but then tended to decline along with output as the recession got under way. Similarly, low interest rates tended to precede periods of rapid output growth but then rise along with GNP at business cycle peaks.

- An increase in the shift variable α will move the IS curve to the right and leave the LM curve unaffected. The result will be a pro-cyclical movement in both y and r, which is usually thought to be more consistent with the evidence (although as we noted above, the correlation between movements in interest rates and output is weak). To explain the pro-cyclical movement in M we could add the assumption that the monetary authorities “accommodate” the expansion - with higher interest rates following the investment boom, the authorities expand the money supply. So long as this expansion in the money supply is not too large, interest rates can still rise overall. In this scenario, the causality will run from interest rates to money supply changes rather than money supply being the exogenous driving factor.[1]

- We can also examine the effects of changes in exogenous variables using algebra. Collect the equilibrium conditions together to get

$$y = f(r, y) + g(r, y, \alpha) \tag{12}$$

$$L(r, y) = (M/p)^* \tag{13}$$

Now totally differentiate (12) and (13) with respect to α to get the matrix equation:

$$\begin{bmatrix} 1 - f_y - g_y & -f_r - g_r \\ L_y & L_r \end{bmatrix} \begin{bmatrix} \frac{dy}{d\alpha} \\ \frac{dr}{d\alpha} \end{bmatrix} = \begin{bmatrix} g_\alpha \\ 0 \end{bmatrix} \tag{14}$$

The determinant is given by

$$\Delta = (1-f_y-g_y)L_r + (f_r+g_r)L_y < 0 \tag{15}$$

and the solutions for the derivatives by

$$\frac{dy}{d\alpha} = \frac{g_\alpha L_r}{\Delta} > 0 \tag{16}$$

$$\frac{dr}{d\alpha} = -\frac{g_\alpha L_y}{\Delta} > 0 \tag{17}$$

as we noted by examining the IS-LM diagram. Also we have:

---

1. This idea of accommodating money supply expansions has difficulty explaining a *lead* of money supply growth over movements in output, but some economists doubt whether this corresponds with the facts. The idea that money supply changes are endogenous accommodating movements would also have difficulty accounting for the evidence of a link between money and output provided by Hume and other economists writing before there was a central bank. The idea of reverse causation from output movements to changes in the money supply has recently regained popularity in the works of the real business cycle theorists.

$$\frac{dc}{d\alpha} = f_y\frac{dy}{d\alpha} + f_r\frac{dr}{d\alpha} \tag{18}$$

and (18) is greater than zero if $|f_y| > |f_r|$ and $|L_r| > |L_y|$ as we shall assume. Also,

$$\frac{di}{d\alpha} = \frac{d}{d\alpha}(y - c) = (1 - f_y)\frac{dy}{d\alpha} - f_r\frac{dr}{d\alpha} > 0 \tag{19}$$

so that consumption and investment both move pro-cyclically in response to an α shock. Furthermore,

$$\frac{d}{d\alpha}(i/y) = y^{-2}\left(y\frac{di}{d\alpha} - i\frac{dy}{d\alpha}\right) = y^{-2}[y(1 - f_y) - i]\frac{dy}{d\alpha} - y^{-1}f_r\frac{dr}{d\alpha} \tag{20}$$

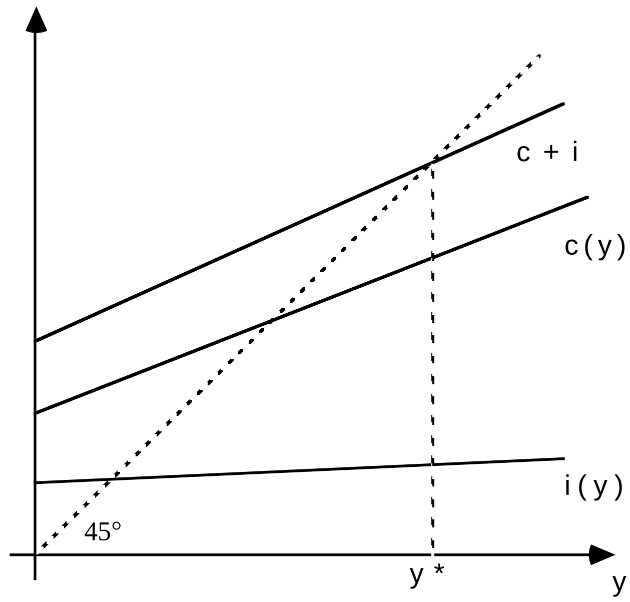
But

$$y(1 - f_y) - i = c - yf_y = c\left[1 - \frac{y\partial f}{c\partial y}\right] > 0 \tag{21}$$

The final inequality in (21) follows from Keynes' "fundamental psychological law". Thus the investment-output *ratio* moves pro-cyclically in response to α shocks as appears to be the case in practice.

3. Dynamics and Stochastic Difference Equations

- An even simpler "Keynesian model" than the IS-LM model we have been discussing assumes nominal interest rates in addition to nominal prices are fixed, takes c and i as functions of y and looks for a level of output y corresponding to equilibrium in the product market. This is the so-called Keynesian Cross model illustrated in the following diagram.



- Using this simplest of models we can see how dynamics is introduced and how that produces a stochastic difference equation for y with cycles in its autocovariances. The same result will carry over to the more complicated models which allow for endogenous variation in interest rates and (at least some) variation in nominal prices.

- When econometricians estimated the consumption function, they found that the error term in a regression of c against y displayed a high degree of serial correlation (that is, estimated values of the error terms were highly correlated with lagged values of themselves). To overcome this problem, they introduced lagged consumption on the right hand side of the regression to get:

$$c_t = \rho c_{t-1} + \alpha y_t + \varepsilon_t \qquad (22)$$

where ε is assumed to be iid. One interpretation of this equation is that there is "inertia" in consumption or savings behavior. Consumption is determined partly by its value last period but partly adjusts to current income. We shall discuss a more satisfactory microeconomic model of this equation later in the course. For the moment, treat it as the result of statistical analysis to get a good "fit" for the consumption function. This was how it originally arose. Using the lag operator, we can write this equation:

$$c_t = \frac{\alpha}{1-\rho L} y_t + \frac{1}{1-\rho L} \varepsilon_t \tag{23}$$

- Dynamics was also introduced into the investment equation. The basic idea used to do this was the “accelerator model” of Samuelson. It is assumed that the required capital stock is proportional to output. The level of investment, or the desired change in the capital stock, will then be proportional to the change in income:

$$K_t \propto y_t \Rightarrow i_t = \beta(y_t - y_{t-1}) + \xi_t \tag{24}$$

where the random error term ξ represents the random exogenous shocks to investment. Again we assume that ξ is iid and uncorrelated at all leads and lags with ε.

- Substituting these equations for c and i into the product market equilibrium condition yields the equation for y:

$$y_t = \frac{\alpha}{1-\rho L} y_t + \beta(y_t - y_{t-1}) + \xi_t + \frac{1}{1-\rho L} \varepsilon_t \tag{25}$$

which, upon multiplying through by $(1-\rho L)$, becomes:

$$(1-\alpha-\beta)y_t = (\rho-\beta\rho-\beta)y_{t-1} + \beta\rho y_{t-2} + \xi_t - \rho\xi_{t-1} + \varepsilon_t \tag{26}$$

- We can obtain some idea of the effect of these dynamic specifications by substituting plausible values of α, β and ρ. Empirically estimated consumption functions will yield $\alpha \cong 0.095$ and $\rho \cong 0.9$ (so the long run marginal propensity to consume is about 0.95), while we shall take as an example $\beta \cong 0.3$. Substituting these values, the stochastic difference equation becomes:

$$0.605\, y_t = 0.33\, y_{t-1} + 0.27\, y_{t-2} + \xi_t - 0.9\, \xi_{t-1} + \varepsilon_t \tag{27}$$

We can find the autocovariances of the stationary distribution for y by squaring (27) and taking expectations of both sides, and then successively multiplying (27) through by y_{t-1} and y_{t-k}, k>2, and in each case taking expectations (note that the mean of y here is zero since the means of ε and ξ are zero - we are thinking of y as deviations around trend). Notice first that by multiplying through by ξ and taking expectations we find

$$\text{cov}(y_t, \xi_t) = \text{cov}(y_{t-1}, \xi_{t-1}) = 1.653\ \sigma_\xi^2 \tag{28}$$

Thus:

$$0.184\ \gamma_0 = 0.178\ \gamma_1 + 0.828\sigma_\xi^2 + \sigma_\varepsilon^2 \tag{29}$$

$$\gamma_0 = 1.015\ \gamma_1 + 4.508\ \sigma_\xi^2 \tag{30}$$

Equations (29) and (30) can be solved for γ_0 and $\gamma_1$as

$$\gamma_0 = 4.245\sigma_\xi^2 + 115.145\ \sigma_\varepsilon^2 \tag{31}$$

$$\gamma_1 = -0.259\sigma_\xi^2 + 113.426\ \sigma_\varepsilon^2 \tag{32}$$

Also, multiplying (27) on both sides by y_{t-k}, k>2 and taking expectations we get the ordinary difference equation:

$$0.605\ \gamma_k = 0.33\ \gamma_{k-1} + 0.27\ \gamma_{k-2} \tag{33}$$

The roots of the characteristic equation corresponding to (33) are:

$$\lambda_1, \lambda_2 = 0.994, -0.449 \tag{34}$$

The solution for the autocovariances will be:

$$\gamma_k = A\lambda_1^k + B\lambda_2^k \tag{35}$$

for constants A and B determined so that γ_0 and γ_1 satisfy the above equations.

- From the solution, we can see that small variance exogenous shocks ε and ξ can be multiplied up into large variance movements in y. Also, since one root is negative and the other close to 1, y can display oscillations and deviations of y from trend can persist for some time. These features of the solution are magnified for larger values of β. For example, for $\beta \cong 0.47$ we find

$$\gamma_0 = 185.7\sigma_\xi^2 + 167.7\sigma_\varepsilon^2 \tag{36}$$

$$\gamma_1 = -181.0\sigma_\xi^2 + 65.4\sigma_\varepsilon^2 \tag{37}$$

$$\lambda_1, \lambda_2 = 0.994, -0.987 \tag{38}$$