Rice University

Mid-Semester Examination Fall 2004

ECON 501: Advanced Microeconomic Theory

Duration: **Ninety (90) minutes**

Permitted Materials: English/Foreign Language Dictionaries and non-programmable calculators

You should answer all questions in Part A and all parts of question 5 in Part B. All questions in Part A are of equal weight, and Part A accounts for half of the total points.

Part A

1. Consider the following binary relation. Take $X = \mathbb{R}^L$, and define $x \succsim y$ if $x \geq y$.

   Show whether or not this relation is complete, transitive, strongly monotone, strictly convex.

2. Cathy's aim is to raise as much money as possible in the T units of time available to her. She is able to allocate her time between two activities x_1 and x_2. Suppose both activities are 'equally productive' in generating money in the sense that if she spends $x \in [0, T]$ units of time in activity $i = 1, 2$, she will raise $f(x)$ dollars. Assume $f(.)$ is a strictly concave and strictly increasing function with $f(0) = 0$.

   State whether you agree or disagree with the following statement and (briefly) explain your reasoning.

   "The solution to Cathy's problem is obvious, she should spend $T/2$ units of time in each activity since that equalizes the amount she raises per unit of time in each activity: that is, if $x_1 = x_2 = T/2$, then we have $f(x_1)/x_1 = f(x_2)/x_2$"

3. *State the Weak Axiom of Revealed Preference and draw a diagram illustrating its implication in a two-good world for the change in consumption of an individual due to a change in prices.*

4. *Explain the meaning of the identity*

$$x_l(p_1, p_2, e(p_1, p_2, u)) \equiv h_l(p_1, p_2, u)$$

   where x_l is the uncompensated demand for good l, h_l is the compensated demand for good l and e is the expenditure function.

   Using this identity derive the Slutsky equation and the 2×2 substitution matrix.

Part B

5. Consider a household that is seen to purchase quantities of just two goods, bread and cheese. Denote quantities of bread by x and quantities of cheese by y. The household comprises two individuals; Andrew, whose preference relation can be represented by the utility function $u_A(x, y) = x$ and Brenda, whose preference relation can be represented by the utility function $u_B(x, y) = y$.

   (a) *Derive the uncompensated demand functions for both Andrew and Brenda and their indirect utility functions.*

   (b) The households' wealth w is divided evenly between Andrew and Brenda. **Suppose that you observe the aggregate demands of this household and you interpret it as if it came from just a single consumer.** ***Find the demands of the supposed single consumer.***

   Recall in lectures that the equivalent variation of a change in prices and income from (p^0, w^0) to (p^1, w^1) is defined as

$$\text{EV} = e\left(p^0, v\left(p^1, w^1\right)\right) - e\left(p^0, v\left(p^0, w^0\right)\right).$$

   If $w^0 = w^1$ and the change in prices are caused by the imposition of commodity taxes then the deadweight loss (DWL) or excess burden of the taxes is given by

$$\text{DWL} = -\text{EV} - \sum_{l=1}^{L} t_l x_l\left(p^1, w^0\right),$$

   where $t_l = p_l^1 - p_l^0$.

   (c) *Briefly explain why this measure may be viewed as a deadweight loss to (social) economic efficiency*

   (d) Suppose that the household initially faces prices $p^0 = (1, 2)$ and has wealth $w^0 = 300$. Then a specific tax of 2 is imposed on bread (i.e. good x) that leads to its price rising to 3 (with the price of cheese, i.e. good y, and the household's wealth both remaining unchanged). *Calculate the DWL under the false assumption that the household demands come from just one consumer.*

   (e) *Using the individuals' indirect utility functions derived in part (a) calculate the two individual dead weight losses,* DWL_A *and* DWL_B*. Explain why* $\text{DWL}_A + \text{DWL}_B$ *does or does not equal* DWL.