**Rice University**

*First Semester Mid-Semester Examination 2001*

**ECON501 Advanced Microeconomic Theory**

*Writing Period:* **Three Hours**

*Permitted Materials: English/Foreign Language Dictionaries and non-programmable calculators*

*You should answer question 1 in Part A and* ***two (2)*** *out of three questions in Part B. The points allotted to each part of each question appear in brackets. The total points for the exam is one hundred (100). Question 1 is worth forty (40) points and each question in Part B is worth thirty (30) points.*

# PART A: Answer all Parts of Question 1.

1. [**40 Points**] Consider an economy with two consumers, Andrew and Bridget, and two goods, quantities of which are denoted by $x$ and $y$, respectively. Andrew's preference relation can be represented by the utility function $u^A(x,y) = x - x^2/2 + y$ while Bridget's preference relation can be represented by the utility function $u^B(x,y) = x + y - y^2/2$.

   (a) Derive the uncompensated demand functions for both Andrew and Bridget and hence show that the aggregate demand function is equal to:

$$x\left(p_x, p_y, w^A, w^B\right) = \begin{cases} 1 - \frac{p_x}{p_y} + \frac{w^B}{p_x} & \text{if } p_x \le p_y \\ -\frac{p_y}{p_x}\left(1 - \frac{p_y}{p_x}\right) + \frac{w^B}{p_x} & \text{if } p_x > p_y \end{cases}$$

$$y\left(p_x, p_y, w^A, w^B\right) = \begin{cases} \frac{w^A}{p_y} - \frac{p_x}{p_y}\left(1 - \frac{p_x}{p_y}\right) & \text{if } p_x \le p_y \\ \frac{w^A}{p_y} + \left(1 - \frac{p_y}{p_x}\right) & \text{if } p_x > p_y \end{cases}$$

   Furthermore show that their indirect utility functions are given by:

$$v^A\left(p_x, p_y, w^A\right) = \begin{cases} \frac{1}{2}\left(1 - \frac{p_x}{p_y}\right)^2 + \frac{w^A}{p_y} & \text{if } p_x \le p_y \\ \frac{w^A}{p_y} & \text{if } p_x > p_y \end{cases}$$

$$v^B\left(p_x, p_y, w^B\right) = \begin{cases} \frac{w^B}{p_x} & \text{if } p_x \le p_y \\ \frac{1}{2}\left(1 - \frac{p_y}{p_x}\right)^2 + \frac{w^B}{p_x} & \text{if } p_x > p_y \end{cases}$$

   (15 points).

(b) Suppose now that aggregate wealth is always divided evenly between Andrew and Bridget. That is, $w^A = w^B = w/2$, where $w$ is the aggregate wealth. With this *price-independent* wealth-sharing rule, show that the following representative indirect utility function can 'rationalize' aggregate demand.

$$v(p_x, p_y, w) = \begin{cases} \frac{w}{\sqrt{p_x p_y}} - 2\sqrt{\frac{p_x}{p_y}} + \frac{2}{3}\left(\frac{p_x}{p_y}\right)^{3/2} & \text{if } p_x \leq p_y \\ \frac{w}{\sqrt{p_x p_y}} - 2\sqrt{\frac{p_y}{p_x}} + \frac{2}{3}\left(\frac{p_y}{p_x}\right)^{3/2} & \text{if } p_x > p_y \end{cases}$$

(5 points).

Recall in lectures that the equivalent variation of a change in prices and income from $(p^0, w^0)$ to $(p^1, w^1)$ is defined as

$$\text{EV} = e\left(p^0, v\left(p^1, w^1\right)\right) - e\left(p^0, v\left(p^0, w^0\right)\right).$$

Further recall from lectures that if wealth is unaltered and the change in prices is caused by the imposition of commodity taxes then the deadweight loss (DWL) or excess burden of the taxes is given by

$$\text{DWL} = -\text{EV} - \sum_{l=1}^{L} t_l x_l\left(p^1, w^0\right),$$

where $t_l = p_l^1 - p_l^0$.

(d) Briefly explain why this measure may be viewed as a deadweight loss to (social) economic efficiency (5 points).

(e) Suppose that the initial *aggregate* budget constraint has $p^0 = (1, 1)$ and $w^0 = 2$. Supposing again that aggregate wealth is always shared equally between Andrew and Bridget, use the representative indirect utility function to calculate the DWL of the imposition of a specific tax of 3 on good $y$, that leads to the price of good $y$ rising to 4 (with the price of good $x$ remaining unchanged). (Hint: your answer should be 19/48) (5 points).

(f) Using the individuals' indirect utility functions derived in 1.a) calculate the two individual dead weight losses, $\text{DWL}_A$ and $\text{DWL}_B$. And show that $\text{DWL}_A + \text{DWL}_B$ is *greater than* DWL. What is the economic meaning of this difference? In particular, can we ascribe any *normative* meaning to the measure DWL derived from the represntative indirect utility function? (Hint you should calculate that $\text{DWL}_B = 0$ and $\text{DWL}_A = 3/8$) (10 points).

## PART B: Answer two (2) out of the following three questions.

2. **[30 Points]** Mr. Croam lives for exactly two periods, $t = 0, 1$. Let $c_t \in \mathbb{R}$ denote his consumption in period $t$. Mr Croam's ($t = 0$) preferences over two-period consumption streams are represented given by the function

$$U(c_0, c_1) := u(c_0) + \delta E u(c_1)$$

where $\delta$ is a discount factor, $u$ is an increasing strictly concave utility function, and the $E$ denotes his expectation (at $t = 0$) about events in $t = 1$.

Initially, suppose that there is no uncertainty. Let $w_1 > 0$ be Mr. Croam's income in period 0 and let $w_1 \geq 0$ denote his income in period 1. Mr. Croam can save or borrow. Let $s \in \mathbb{R}$ denote his saving (notice that $s$ could be negative) and let $\rho$ denote the gross return on saving (i.e., $\rho = 1 + r$ if $r$ is the interest rate). Thus, his consumption in period 0 is $w_0 - s$ and his consumption in period 1 is $w_1 + \rho s$. Assume interior solutions throughout.

(a) Write down necessary and sufficient conditions for Mr. Croam's chosen saving $s^*$ to be greater than 0. (10 points).

(b) Suppose that $w_1 = 0$ and that the conditions from part (a) hold. Find a condition on the Mr. Croam's coefficient of relative risk aversion that is necessary and sufficient for $s^*$ to be (locally) increasing in $\rho$. (10 points).

Now suppose that Mr. Croam faces uncertainty over his period 1 income. Specifically, suppose that his period 1 income is given by $w_1 + \tilde{x}$ where $w_1 \geq 0$ and $E\tilde{x} = 0$. Let $s^{**}$ denote Mr. Croam's new optimal saving.

(c) Show that if $u''' > 0$, then $s^{**} > s^*$ [Hint: Suppose that $s^{**} = s^*$ and compare the first order conditions.] (10 points).

3. **[30 Points]** Consider a two-firm Cournot (quantity-competition) model with constant returns to scale but in which the firms' costs differ. Let $c_j$ denote the firm $j$'s cost per unit of output produced and assume $c_1 < c_2$. Let aggregate demand be given by $Q = 1 - p$.

(a) Derive the Nash equilibrium of this model. Under what conditions does it involve only one firm producing? Which will it be? (10 points).

(b) Show that if more than one firm is making positive sales show that we cannot have productive efficiency. In this situation what is the correct measure of (Marshallian) welfare loss relative to a fully efficient (that is, perfectly competitive) outcome. (10 points).

(c) Calculate the rate at which the Marshallian welfare changes as $c_1$ (respectively, $c_2$) changes. Can it ever be the case that a reduction in one of the firm's marginal cost *reduces* the Marshallian welfare in this market? (10 points).

4. **[30 Points]** A firm's production of bacon generates a smelly gas as an unpleasant side product. Let $c(y, m; \mathbf{w})$ denote the (minimum) input cost of producing $y$ tons of bacon and $m$ cubic meters of gas when input prices are given by the vector $\mathbf{w} \gg \mathbf{0}$. Let $p > 0$ denote the price of bacon. Assume that $\partial c/\partial y > 0$, $\partial c/\partial m < 0$ and that $c(.,.;\mathbf{w})$ is strictly convex in $y$ and $m$. Let stars $^*$ denote solutions and assume throughout that $y^* > 0$.

   (a) Show that $c(y, m; .)$ is concave in $\mathbf{w}$. (10 points).

   (b) Suppose that the government imposes a ceiling on gas emissions such that $m \leq \bar{m}$. Assuming that this constraint binds, write down the firm's profit maximization problem with respect to $y$, and find necessary and sufficient conditions for $\partial y^*/\partial \bar{m} > 0$. (10 points).

   (c) Suppose now that the government abandons its emissions ceiling and replaces it with a tax $t > 0$ on gas emissions. Thus, the new cost of producing $(y, m)$ is given by $c(y, m; \mathbf{w}) + tm$. Write down the firm's profit maximization problem with respect to $y$ and $m$. Show that maximized profits are convex in $t$, and that $\partial m^*/\partial t \leq 0$. (10 points).