ECON501 Advanced Microeconomic Theory 1
Fall Semester 2007
Problem Set 5

The due date for this problem set is MONDAY Nov 12

1. **(Polak)** The following utility function (sometimes called Dixit-Stiglitz preferences) is often used in international trade applications:

$$u(x_0, x_1, \ldots, x_L) = x_0^{1-\alpha} \left(\sum_{\ell=1}^{L} x_\ell^\beta \right)^{\alpha/\beta}$$

where α and β are both in the open interval $(0, 1)$. In the following assume $p \gg 0$ and $w > 0$. Let $G := \{1, \ldots, L\}$.

   (a) Show that the proportion of wealth spent on x_0 is $(1 - \alpha)$.

   (b) Verify that we can express the Marshallian demand for each good ℓ in G as a function just of prices $(p_\ell)_{\ell \in G}$ and the expenditure on those goods w_G.

   (c) Write down the expenditure function corresponding to an appropriately chosen subutility function for the goods in G.

   (d) Consider breaking the consumer's maximization problem into a two-step budget process, where the first stage is to allocate expenditure between good 0 and the group of goods in G. Show that we can write the first-stage of the consumer's maximization problem as

$$\max_{\langle x_0, X_G \rangle} U(x_0, X_G) \text{ subject to } p_0 x_0 + P_G X_G \leq w$$

   where $X_G := \left(\sum_{\ell=1}^{L} x_\ell^\beta \right)^{1/\beta}$ and P_G is a scaler that is a function just of the prices $(p_\ell)_{\ell \in G}$.

2. **(Polak) Additive Separability.** In class we discussed weak separability. A stronger assumption is additive separability. Preferences are said to be additively separable if they can be represented by an additively separable utility function:

$$u(x_1, \ldots, x_L) = \sum_{\ell=1}^{L} u_\ell(x_\ell),$$

where each x_ℓ here refers to a single good, not a group. Such representations are very tractable, but additive separability is very restrictive.

(a) Show that CES preferences are additively separable. Show that Stone-Geary preferences

$$u(x_1, x_2) = (x_1 - c_1)^{\alpha} (x_2 - c_2)^{1-\alpha}$$

are additively separable.

(b) Suppose there are just two goods and that preferences are additively separable. Suppose that $(1, 2) \sim (2, 1)$ and that $(1, 4) \sim (2, 2) \sim (5, 1)$. Find two other points that are indifferent to each other.

(c) Return to the $L-$good case. If the sub-utility functions $u_\ell(.)$ are increasing and strictly concave for all ℓ, show that the wealth effects $\partial x_\ell(p, w)/\partial w > 0$ for all w.

(d) Argue that, if preferences are additively separable, then they are weakly separable for *any* groupings of the commodities that we might choose.

(e) Under the same assumptions as part (d), show that every pair of goods are (Hicks) net substitutes. [Hint: use your answers to (c) and (d) and the discussion in class of the implications of weak separability on the substitution effect s_{ij} between two goods in different groups.]

3. Consider an economy with two consumers, Abigail and Brian, and two goods, denoted x and y. Abigail's preference relation can be represented by the utility function $u_A(x, y) = x$ while Brian's preference relation can be represented by the utility function $u_B(x, y) = \log(x) + y$.

(a) Derive the uncompensated demand functions for both Abigail and Brian and show that their indirect utility functions are

$$v_A(p_x, p_y, w_A) = w_A/p_x, \text{ for Abigail}$$

and

$$v_B(p_x, p_y, w_B) = \begin{cases} \log(p_y/p_x) + w_B/p_y - 1 & \text{if } w_B > p_y \\ \log(w_B/p_x) & \text{if } w_B \leq p_y \end{cases}, \text{ for Brian.}$$

(b) Given that aggregate wealth in this economy is divided evenly between Abigail and Brian show that the following representative indirect utility function can generate aggregate demand for the case where $w/2 > p_y$.

$$v(p_x, p_y, w) = \frac{w}{\sqrt{p_x p_y}} + 2\sqrt{\frac{p_y}{p_x}}$$

Recall in lectures that the equivalent variation of a change in prices and income from (p^0, w^0) to (p^1, w^1) is defined as

$$\text{EV} = e\left(p^0, v\left(p^1, w^1\right)\right) - e\left(p^0, v\left(p^0, w^0\right)\right).$$

(c) If $\text{EV} > 0$, what does this signify and why?

If $w^0 = w^1$ and the change in prices are caused by the imposition of commodity taxes then the deadweight loss (DWL) or excess burden of the taxes is given by

$$\text{DWL} = -\text{EV} - \sum_{l=1}^{L} t_l x_l\left(p^1, w^0\right),$$

where $t_l = p_l^1 - p_l^0$.

(d) Briefly explain why this measure may be viewed as a deadweight loss to (social) economic efficiency.

(e) Suppose that the initial aggregate budget constraint has $p^0 = (1, 1)$ and $w^0 = 40$. Using the representative indirect utility function calculate the DWL of the imposition of a specific tax of 3 on good y, that leads to the price of good y rising to 4 (with the price of good x remaining unchanged).

(f) Using the individuals' indirect utility functions derived in 3a) calculate the two individual dead weight losses, DWL_A and DWL_B. Explain why $\text{DWL}_A + \text{DWL}_B$ does or does not equal DWL.