

ECON501 Advanced Microeconomic Theory 1
Fall Semester 2007
Problem Set 4

The due date for this problem set is Friday November 2

1. EV and CV measures for several changes.

Consider the following expenditure function:

$$e(p, u) = \frac{u}{(p_1^{-1} + p_2^{-1})}$$

- (a) Derive the money metric compensation function.
- (b) Evaluate the EV and CV for the changes
 - i. from $(p^0, w^0) = (1, 1, 1)$ to $(p^1, w^1) = (1, 1, 2)$
 - ii. from $(p^0, w^0) = (1, 1, 1)$ to $(p^2, w^2) = (2, 1, 5/2)$.

Which is the preferred change and why?

2. EV, CV and Consumer Surplus in several markets.

Suppose that a consumer's consumption set is \mathbb{R}_+^L and that her demand for good l is given by the uncompensated demand function $x_l(p, w) = \alpha_l w / p_l$, where $\alpha_l > 0$ and $\sum_l \alpha_l = 1$.

- (a) Derive her indirect utility function and expenditure function and show that her money metric utility function is

$$e(\bar{p}, v(p, w)) = \frac{\bar{p}^{\alpha_1} \bar{p}^{\alpha_2} \dots \bar{p}^{\alpha_L}}{p^{\alpha_1} p^{\alpha_2} \dots p^{\alpha_L}} w.$$

Recall in lectures that the equivalent variation of a change in prices and income from (p^0, w^0) to (p^1, w^1) is defined as

$$EV = e(p^0, v(p^1, w^1)) - e(p^0, v(p^0, w^0)).$$

If $w^0 = w^1$ and the change in prices are caused by the imposition of commodity taxes then the deadweight loss or excess burden of the taxes is given by

$$DWL = -EV - \sum_{l=1}^L t_l x_l(p^1, w^0),$$

where $t_l = p_l^1 - p_l^0$.

- (b) Suppose that for our consumer $p^0 = (1, 1, \dots, 1)$ and $w^0 = 1$. Calculate the DWL of the imposition of a specific tax of 1 on good 1, that leads its price rising to 2 (and all other prices remaining unchanged) given that $\alpha_1 = 2/5$ for the consumer.
- (c) Using the uncompensated demand function for good 1, calculate the approximate measure for the DWL

$$\begin{aligned}
 DWL &\approx -CS - \sum_{l=1}^L t_l x_l(p^1, w^0) \\
 &= \int_{p_1=1}^{p_1=2} x_1(p_1, 1) dp_1 - x_1(2, 1) \\
 &= \int_{p_1=1}^{p_1=2} \frac{2}{5p_1} dp_1 - \frac{2}{5 \times 2}
 \end{aligned}$$

- (d) Calculate the consumer surplus based approximate measure of the DWL for an imposition of specific taxes of 1 on both goods 1 and 2, that leads the prices of both goods rising to 2 (and all other prices remaining unchanged) given that $\alpha_2 = 1/5$. Illustrate this approximate measure for the DWL in two demand-supply diagrams, one for each good that is taxed.
- (e) Calculate the exact measure (based on the EV of the price change) of the DWL for our consumer that arises when both goods 1 and 2 have a specific tax of 1 levied on them. If you have calculated it correctly, you should find it is LESS than the DWL that arose when only good 1 was taxed. How can this answer be reconciled with the measure you calculated in the previous part (d)?