

ECON501 Advanced Microeconomic Theory 1
Fall Semester 2007
Problem Set 3

The due date for this problem set is **Friday October 26**.

1.

- (a) Derive the expenditure function $e(p_1, p_2, u)$ from preferences represented by the utility function $u(x_1, x_2) = x_1^2 + x_2^2$ where $x_1 \geq 0$ and $x_2 \geq 0$.

- (b) For this expenditure function derive and draw the set

$$V_4 = \{(x_1, x_2) \in \mathbb{R}_+^2 : p_1x_1 + p_2x_2 \geq e(p_1, p_2, 4) \text{ for all } p_1 > 0, p_2 > 0\}$$

- (c) Draw on the same picture the upper contour set

$$\mathcal{S}_4 = \{(x_1, x_2) \in \mathbb{R}_+^2 : x_1^2 + x_2^2 \geq 4\}$$

- (d) Explain the relationship between V_4 and \mathcal{S}_4 , in particular explain why are they not equal.

- (e) Without further calculation, write down the expenditure function $\hat{e}(p_1, p_2, u)$ derived from preferences represented by the utility function $\hat{u}(x_1, x_2) = x_1^3 + x_2^3$ and also draw the set

$$\hat{V}_8 = \{(x_1, x_2) \in \mathbb{R}_+^2 : p_1x_1 + p_2x_2 \geq \hat{e}(p_1, p_2, 8) \text{ for all } p_1 > 0, p_2 > 0\}$$

2. You are given the following partial information about the purchases of two individuals, Brian who lives in Australia and Atsushi who lives in Japan. Assume the consumption set for both is \mathbb{R}_+^2 , that is, they only consume goods x and y .

	Brian		Atsushi	
	Quantity	Price	Quantity	Price
Commodity x	100	\$1	120	100 yen
Commodity y	100	\$1	?	80 yen

Over what range of quantities of y consumed by Atsushi could you conclude:

- (a) that Atsushi would be better off if he had the same wealth as Brian and faced the same prices as Brian does in Australia?

- (b) that Brian would be better off if he had the same wealth as Atsushi and faced the same prices as Atsushi does in Japan?
- (c) that the behavior of the two consumers is inconsistent with both having the same preferences?
- (d) that there is insufficient evidence to justify any of the conclusions (a), (b) or (c)?

For the rest of the question assume that Atsushi and Brian have the same preferences.

Over what range of quantities of y consumed by Atsushi would you conclude:

- (e) that x is an inferior good for these two consumers?
- (f) that y is an inferior good for these two consumers?

3. There are two commodities. For the budget described by the prices $p^1 = (1, 1)$ and wealth $w^1 = 8$, the consumer is observed to purchase $x^1 = (4, 4)$, while for the prices $p^2 = (1, 4)$ and wealth $w^2 = 26$ the consumer is observed to purchase a consumption bundle x^2 that costs 26.

- (a) Determine the region of permissible choices for x^2 consistent with maximization of preferences.
- (b) Determine the region of permissible choices for x^2 consistent with maximization of preferences representable by a utility function $u(x_1, x_2) = x_1 + v(x_2)$ where v is a strictly concave function.
- (c) Determine the region of permissible choices for x^2 consistent with maximization of preferences representable by a utility function $u(x_1, x_2) = v(x_1) + x_2$ where v is a strictly concave function.
- (d) Determine the region of permissible choices for x^2 consistent with maximization of preferences for which both goods are normal.
- (e) Determine the region of permissible choices for x^2 consistent with maximization of homothetic preferences.

Remark: To answer this question I suggest using good pictures as much as possible.

Question 4 follows on the next page.

4. George lives in a world where there are only three goods. You observe George's consumption on two occasions. On the first occasion, the price vector is $(\bar{p}_1, \bar{p}_2, \bar{p}_3) = (1, 1, 1)$ and you observe George choosing $(\bar{x}_1, \bar{x}_2, \bar{x}_3) = (6, 18, 12)$. On the second occasion, the price vector is $(\hat{p}_1, \hat{p}_2, \hat{p}_3) = (1, 3, 2)$ and you observe George choosing $\hat{x}_3 = 8$. Unfortunately, you were not fully paying attention and you do not observe \hat{x}_1 or \hat{x}_2 . You also do not directly observe George's wealths \bar{w} and \hat{w} corresponding to these two choices.
- (a) Find the set of values of (\hat{x}_1, \hat{x}_2) that are consistent with the weak axiom of revealed preference. Make a rough drawing of the set.
- (b) Now suppose that you know that George is maximizing a Cobb-Douglas utility function $u(x_1, x_2, x_3) = x_1^{\alpha_1} x_2^{\alpha_2} x_3^{\alpha_3}$. Find (\hat{x}_1, \hat{x}_2) .