ECON501 Advanced Microeconomic Theory 1
Fall Semester 2007
Problem Set 2

The due date for this problem set is Friday October 19.

1. Solve the UMP of a consumer whose preference relation over the consumption set $X = \mathbb{R}^3_+$ can be represented by the utility function.:
$$u\left(x_1, x_2, x_3\right) = A x_1^{\alpha} x_2^{\beta} x_3^{1-\alpha-\beta}$$
where $A > 0$, and α, β and $\alpha + \beta$ are all in $(0, 1)$. Find the indirect utility function, the minimum expenditure function, and the Hicksian demand function.

2. Let $x\left(p, w\right) = \left(x_1\left(p, w\right), x_2\left(p, w\right), \ldots, x_L\left(p, w\right)\right)$ be a demand function.

   (a) Show that $\sum_{k=1}^{L} p_k \frac{\partial}{\partial w} x_k = 1$.

   (b) $-\frac{\partial}{\partial p_l} x_l\left(p, w\right) / \left(\frac{x_l}{p_l}\right)$ is called the price elasticity of demand for good l (with respect to its own price). Show that the consumer will spend less on good l as p_l (marginally) goes up if and only if the price elasticity is more than one.

   (c) If $\frac{\partial}{\partial w} x_l\left(p, w\right) \geq 0$, good l is said to be normal at (p, w). A good is called a normal good if it is normal at any (p, w), otherwise it is called an inferior good. If $\frac{\partial}{\partial p_l} x_l\left(p, w\right) > 0$ occurs, good l is called a Giffen good. Show that a Giffen good cannot be normal. Show graphically that an inferior good is not necessarily a Giffen good.

3. Homothetic preferences can be represented by a utility function that is homogeneous of degree one. That is, $u\left(\gamma x\right) = \gamma u\left(x\right)$ for all x in the consumption set and all $\gamma > 0$, such that γx is in the consumption set. Assuming an interior solution for the consumer's UMP, show that for *this* representation of homothetic preferences:

   (a) the uncompensated system of demand functions, $x\left(p, w\right)$ and the indirect utility function, $v\left(p, w\right)$ are homogeneous of degree one in w. That is, $x\left(p, w\right) \equiv w x\left(p, 1\right)$ (or equivalently, $x_l\left(p, w\right) \equiv w x_l\left(p, 1\right)$ for all $l = 1, \ldots, L$), and $v\left(p, w\right) \equiv w v\left(p, 1\right)$.

   (b) that the income elasticity of demand for each good is one.

(c) the expenditure function, $e(p, u)$, and the compensated (Hicksian) system of demand functions, $h(p, u)$, are homogeneous of degree one in u.

(Hint for a): assuming an interior solution (i.e. $x_l > 0$ for all $l = 1, \ldots, L$), show from the first order conditions of the UMP that the Lagrange multiplier for the budget constraint, $\lambda = \sum_{l=1}^{L} u_l(x) x_l / w$. Apply Euler's theorem [see MWG Theorem M.B.2 p929] to show that $\lambda = u(x)/w$. Go back to the first order necessary conditions and show that $x_l(p, w) = w x_l(p, 1)$ also satisfies these conditions.

Hint for c): exploit the identities linking expenditure and indirect utility functions.)

4. The matrix below records the demand substitution effects for a consumer consuming three goods at the prices $p_1 = 1$, $p_2 = 2$ and $p_3 = 6$.

$$\begin{bmatrix} -10 & ? & ? \\ ? & -4 & ? \\ 3 & ? & ? \end{bmatrix}$$

Supply the missing numbers. Does the resulting matrix possess all the properties of a substitution matrix?

(a) A consumer spends a budget of \$10 entirely on two goods whose prices are $p_1 = \$1$ and $p_2 = \$2$. Given that she purchases two units of good one, that her marginal propensity to consume good one is $1/2$ and that the own price elasticity of her (uncompensated) demand for good one is -2.5, deduce the value of the own price elasticity of her uncompensated demand for good two.

(b) What is Roy's identity? If the value taken by the marginal utility of income of the consumer in a) is 2, what are the values of the derivatives of her indirect utility function with respect to prices?