ECON501 Advanced Microeconomic Theory 1
Fall Semester 2007
Problem Set 1

The due date for this problem set is Friday October 12.

1. (Polak) **Social Preferences.** Consider a social planner who has preferences over 'welfare' vectors $\mathbf{w} \in \mathbb{R}_+^N$, with the interpretation that the i^{th} component w_i is the welfare of the i^{th} person in society. Suppose that the social planner's *only* concern is to maximize the welfare of the least well-off person in society; she does not care about anything else.

   (a) Write down a welfare function $W : \mathbb{R}_+^N \to \mathbb{R}$ that represents this social planner's preference relation over welfare vectors. Explain briefly whether or not this preference relation is continuous, monotone, strongly monotone, convex or strictly convex. It may help to sketch the indifference map.

   (b) How are monotone and strict monotone here related to Pareto critieria?

   (c) Now suppose the planner decides that, if the least well-off person in $\mathbf{w}$ and $\mathbf{w}'$ are equally well off, then she prefers $\mathbf{w}$ to $\mathbf{w}'$ if the second-least worst off person in $\mathbf{w}$ is better off than the second-least worst off person in $\mathbf{w}'$, and if this is a tie then she shifts her concern to the third-worst off person, et cetera. Explain whether or not this preference relation is continuous, monotone, strongly monotone, convex or strictly convex.

2. Draw indifference curves of a preference relation over two goods (denoted 1 and 2) which lead to the following property for a consumer's uncompensated demand for good 1:

   (a) quantity demanded of good 1 is zero at some price vector.

   (b) the demand curve for good 1 is perfectly own price elastic at some price vector.

   (c) the demand curve for good 1 is discontinuous at some price vector.

   For the above three cases provide a condition on the preferences *sufficient* to rule out the phenomena in question.

3. Let the consumption set $X = \mathbb{R}^2_+$ and assume that a preference relation $\succsim$ satisfies the following three properties:

   ADD: $(x_1, x_2) \succsim (x'_1, x'_2)$ implies that $(x_1 + a, x_2 + b) \succsim (x'_1 + a, x'_2 + b)$ for all a and b.

   MON: If $x_1 \geq x'_1$ and $x_2 \geq x'_2$ then $(x_1, x_2) \succsim (x'_1, x'_2)$; in addition, if either $x_1 > x'_1$ or $x_2 > x'_2$ then $(x_1, x_2) \succ (x'_1, x'_2)$.

   CON: Continuity.

   (a) Show that if $\succsim$ has a linear representation (that is, $\succsim$ can be represented by a utility function $u(x_1, x_2) = \alpha x_1 + \beta x_2$ with $\alpha > 0$ and $\beta > 0$), then $\succsim$ satisfies *ADD*, *MON* and *CON*.

   (b) Show that the three properties are necessary for $\succsim$ to have a linear representation. Namely, show that for any pair of the three properties, there is a preference relation that does not satisfy the third property.