1. CONSUMER THEORY & DEMAND

1.1 Consumer Choice Theory

Four building blocks

1. set of alternatives – Consumption Set X
2. a binary relation – Preferences

   $$\succsim \;\subset X \times X, \text{ i.e. } (x, y) \in \;\succsim\; \Leftrightarrow x \succsim y$$

   represented by a utility function

   $$x \succsim y \Leftrightarrow u(x) \geq u(y)$$

3. feasible set – Walrasian (or Competitive) Budget Set

   $$\mathcal{B}_{p,w} \subset X$$

4. a behaviorial assumption
   – Preference (i.e. Utility) Maximization

1.1.1 Consumption Set

(see MWG 2.C pp18-20)

- Finite number of commodities
  - divisible or indivisible
- no time dimension, stocks rather than flows
  - time (or location) can be built into definition.
  - uncertainty

Commodity Space — $\mathbb{R}^L$

Consumption Set $X \subset \mathbb{R}^L$

1.1.2 Preferences

(see MWG 3.B-C)

Can derive from $\succsim$ two other relations:

1. Strict Preference

$$x \succ y \;\Leftrightarrow\; x \succsim y \text{ and NOT} \,(y \succsim x)$$

2. Indifference

$$x \sim y \;\Leftrightarrow\; x \succsim y \text{ and } y \succsim x$$

Rationality Assumptions

(i) *completeness*: for all $x, y \in X$

$$\text{either } x \succsim y \text{ or } y \succsim x \text{ (or both)}$$

(ii) *transitivity*: for all $x, y, z \in X$

$$\text{if } x \succsim y \text{ and } y \succsim z \text{ then } x \succsim z$$

A preference relation that satisfies completeness and transitivity is often referred to as a *rational* preference relation or a *preference ordering.*

Desirability Assumptions

(iii) *monotonicity*: if $y \gg x$ then $y \succ x$

- implicit assumption of free disposal
- rules out satiation of *all* commodities

(iii') *strong monotonicity*: if $y \geq x$ and $y \neq x$ then $y \succ x$

(iii'') *local non-satiation*: for all $x \in X$ and all $\varepsilon > 0$ there exists y such that

$$|y - x| < \varepsilon \text{ and } y \succ x$$

- rules out "thick" indifference curves.

Convexity Assumptions

Let x be in X. Define:

1. $\succsim^x \equiv \{y \in X \mid y \succsim x\}$ "at least as good as set"
2. $\precsim^x \equiv \{y \in X \mid x \succsim y\}$ "no better than set"
3. $\succ^x \equiv \{y \in X \mid y \succ x\}$ "better than set"
4. $\prec^x \equiv \{y \in X \mid x \prec y\}$ "worse than set"
5. $\sim^x \equiv \{y \in X \mid x \sim y\}$ "indifference set"

(iv) *convexity*: for all x in X, the "at least as good as set", $\succsim^x$, is convex.

That is,

$$\text{if } y \succsim x \text{ and } z \succsim x \text{ then } \alpha y + (1 - \alpha) z \succsim x$$

for *any* α in $(0, 1)$

- decreasing relative marginal satisfaction for any two commodities
- formal expression of primitive inclination of economic agent's preference for diversification

Exercise: Give an economically interpretable example with two commodities where preferences are *not* convex.

(iv) *strict convexity*: for all x, y, z in X, with $y \neq z$

$$\text{if } y \succsim x \text{ and } z \succsim x \text{ then } \alpha y + (1-\alpha) z \succ x$$

for *any* α in $(0, 1)$.

Representation

A (quasi-concave) function $u : X \to \mathbb{R}$ always generates a (convex) preference ordering.

$$\begin{aligned} \text{i.e. } u(x) &\geq u(y) \Rightarrow x \succsim y \\ (\ u(\alpha x + (1-\alpha) y) &\geq \min\{u(x), u(y)\}\ ) \end{aligned}$$

Obs: Suppose $u(\cdot)$ generates $\succsim$, then for *any* increasing function $T : \mathbb{R} \to \mathbb{R}$, $V(x) \equiv T(u(x))$ also generates the *same* preference ordering.

$$\text{i.e. } u(x) \geq u(y) \Leftrightarrow x \succsim y \Leftrightarrow V(x) \geq V(y)$$

- Properties of utility functions which are *invariant* to increasing transformations are called *ordinal*.
  - e.g. convexity, monotonicity

When can preferences be represented by a utility fn?

Obs: Even with every assumption made so far, a preference ordering need not be representable by a utility function.

Example: Lexiographic Ordering.

Take $L = 2$ and define

$$x \succsim y \quad \text{if either} \quad x_1 > y_1 \quad \text{or} \quad x_1 = y_1 \text{ and } x_2 \geq y_2$$

Exercise: Verify the lexiographic ordering satisfies strong monotonicity and strict convexity.

Continuity Assumption

(v) Preference relation $\succsim$ is *continuous* if the "at-least-as-good-as" relation $\succsim$ is preserved under limits. That is, for any sequence $(x^n, y^n)_{n=1}^{\infty}$ for which $x^n \to x$ and $y^n \to y$

if for all n, $x^n \succsim y^n$, then $x \succsim y$.

Obs: Above definition is equivalent to requiring $\succsim^x$ and $\precsim^x$ are both closed and $\succ^x$ and $\prec^x$ are both open for all $x \in X$.

PUNCHLINE

Theorem 1: Following are equivalent for a rational preference relation $\succsim$.

(i) $\succsim$ is continuous.

(ii) There exists a *continuous* $u(\cdot)$ that represents $\succsim$.

Obs: Result is true in considerable generality.

For monotonic preferences there is simple proof.

Theorem 2: Let $\succsim$ be a rational and continuous preference relation represented by $u(\cdot)$. Then:

1. $u(\cdot)$ is (strictly) increasing $\Leftrightarrow$ $\succsim$ satisfies (strong) monotonicity.
2. $u(\cdot)$ is (strictly) quasiconcave $\Leftrightarrow$ $\succsim$ is (strictly) convex.

Recall:

1. strict quasi-concavity of $u(\cdot)$ requires
$$u(y) \geq u(x) \Rightarrow u(\alpha x + (1-\alpha) y) > u(x) \text{ for any } \alpha \in (0,1).$$
2. strict convexity of $\succsim$ requires
$$y \succsim x \Rightarrow \alpha x + (1-\alpha) y \succ x \text{ for any } \alpha \in (0,1).$$

1.1.3 Feasible Set - Walrasian (Competitive) Budget Set

$$p = \begin{pmatrix} p_1 \\ \vdots \\ p_L \end{pmatrix} \in \mathbb{R}^L \text{ (usually assume } p_\ell > 0)$$

- prices quoted publicly
  - principle of *completeness* or *universality* of markets
- prices beyond influence of any particular economic agent
  - price-taking hypothesis

Consumer faces two types of constraints

- physical or environmental
  - embodied in definition of consumption set X
- economic constraints
  - we abstract from all but two

1. price vector p, given by market
2. agent's wealth $w > 0$

$$\mathcal{B}_{p,w} = \{x \in X \mid p.x \equiv p_1 x_1 + \ldots + p_L x_L \leq w\}$$

1.1.4 Behavioral Assumption:- Preference Maximization

$$\begin{aligned} & \mathcal{C}\left(\mathcal{B}_{p,w}\right) \equiv x\left(p,w\right) \\ \equiv \quad & \{x \in \mathcal{B}_{p,w} \mid x \succsim y \text{ for all } y \in \mathcal{B}_{p,w}\} \end{aligned}$$