4.5 Externalities and Public Goods

Ref: MWG Chapter 11.

FFWT any competitive equil. is Pareto optimal

SFWT (given suitable convexity assumptions) *any* Pareto optimal allocation can be supported as a competitive equil.

Tends to suggest possibilities for welfare-enhancing intervention in mktplace can be strictly limited to carrying out wealth transfers for purposes of achieving distributional aims.

Market Failures

	$\nearrow$	consumption side	– noise pollution
1) *externalities*	$\rightarrow$	production side	– chemical plant's discharges reducing fishery's catch
2) *public goods*	$\rightarrow$	non-rivalry in consumption	– national defence
	$\searrow$		– flood control
		non-excludable	– lighthouse

DEFN. An *externality* is present whenever the well-being of a consumer or the production possibilities of a firm are *directly* affected by the actions of another agent in the economy.

c.f. *"Pecuniary"* externality.

$\surd$ Fishery's productivity affected by emissions from oil refinery.

$\times$ Fishery's profitability affected by price of oil.

Latter is *mediated* by *prices* through mkts
and outcome in competitive mkts is Pareto optimal/efficient.

Example of Bilateral Externality

Consumer 1 chooses consumption bundle x^1 and some action $h \in \mathbb{R}_+$

UMPs $v_1\left(p, w^1, h\right) = \max_{\{x^1 \geq 0,\ h \geq 0\}} u^1\left(x^1, h\right)$ s.t. $p.x^1 \leq w^1$.

$v_2\left(p, w^2, h\right) = \max_{\{x^2 \geq 0\}} u^2\left(x^2, h\right)$ s.t. $p.x^2 \leq w^2$.

Simplify exposition by supposing preferences are additively separable between consumption and h,

$$\begin{aligned} v_1\left(p, w^1, h\right) &= \phi_1(h) \quad \left(+\, \hat{v}_1\left(p, w^1\right)\right) \\ v_2\left(p, w^2, h\right) &= \phi_2(h) \quad \left(+\, \hat{v}_2\left(p, w^2\right)\right) \end{aligned}$$

Efficient outcome: $\max_{h \geq 0} \phi_1(h) + \phi_2(h)$

FONC: $\phi_1'\left(h^0\right) \leq -\phi_2'\left(h^0\right)$ (with equality if $h^0 > 0$)

Equilibrium outcome: Consumer 1: $\max_{h \geq 0} \phi_1(h)$

FONC: $\phi_1'(h^*) \leq 0$ (with equality if $h^* > 0$)

Pigouvian Taxation

For negative externality set tax $t_h = -\phi_2'\left(h^0\right)$

Consumer 1's pblm becomes : $\max_{h\geq 0} \phi_1(h) - t_h h$

FONC: $\phi_1'\left(\hat{h}\right) \leq t_h$ (with equality if $\hat{h} > 0$)

Notice $\hat{h} = h^0$, optimality-restoring tax is exactly equal to *marginal externality* at the optimal solution.

Positive Externality

Govt sets *subsidy* $s_h = \phi_2'\left(h^0\right) > 0$

Consumer 1's pblm becomes : $\max_{h\geq 0} \phi_1(h) + s_h h$

FONC: $\phi_1'\left(\hat{h}\right) + s_h \leq 0$ (with equality if $\hat{h} > 0$)

Coasean Critique

- Coase argued:
  1. If there are no transactions costs of bargaining, then the Pigouvian solution is *wrong.*
  2. If there are transactions costs of bargaining, then the Pigouvian solution is incomplete.
- Bargaining between two parties → Pareto efficient outcome (irrespective of who has property rights).

a) Assign right to "externality-free" environment to consumer 2.

Consumer 2 can make "take-it-or-leave-it" offer, (h, T) to consumer 1.

Consumer 2's pblm $\max_{h\geq 0, T} \phi_2(h) + T$ s.t. $\phi_1(h) - T \geq \phi_1(0)$

$\Rightarrow T = \phi_1(h) - \phi_1(0)$

So 2's pblm may be expressed

$$\max_{h\geq 0} \phi_2(h) + \phi_1(h) - \phi_1(0)$$

$$\text{FONC: } \phi_1'\left(h^0\right) \leq -\phi_2'\left(h^0\right) \text{ (with equality if } h^0 > 0)$$

I.e. socially optimal outcome.

b) Assign right "to pollute" to consumer 1.

Consumer 1 can make "take-it-or-leave-it" offer, (h, T), to consumer 2.

Consumer 1's pblm $\max_{h\geq 0,T} \phi_1(h) + T$ s.t. $\phi_2(h) - T \geq \phi_2(h^*)$

$$\Rightarrow T = \phi_2(h) - \phi_2(h^*)$$

So 1's pblm may be expressed

$$\max_{h\geq 0} \phi_1(h) + \phi_2(h) - \phi_2(h^*)$$

And once again,

$$\text{FONC: } \phi_1'\left(h^0\right) \leq -\phi_2'\left(h^0\right) \text{ (with equality if } h^0 > 0)$$

Pigouvian Tax when 2 assigned right to "externality-free" environment.

$$\max_{h\geq 0,T} \phi_2(h) + T$$

$$\text{s.t. } \phi_1(h) - t_h h - T \geq \phi_1(0)$$

$$\text{i.e. } T = \phi_1(h) - \phi_1(0) - t_h h$$

So 2' pblm may be expressed

$$\max_{h\geq 0} \phi_2(h) + \phi_1(h) - t_h h - \phi_1(0)$$

$$\text{FONC: } \phi_1'\left(\hat{h}\right) + \phi_2'\left(\hat{h}\right) - t_h \leq 0 \text{ (with equality if } h^0 > 0)$$

$$\Rightarrow \phi_1'\left(\hat{h}\right) = -\phi_2'\left(\hat{h}\right) - \phi_2'\left(h^0\right)$$

Public Goods

DEFN: A *public good* is a commodity for which use of a unit of the good by one agent does not preclude its use by other agents.

[i.e. non-depletable, non-rivalrous in consumption].

Distinction

Excludable – e.g. patent

Non-excludable – e.g. national defense, flood control

Conditions for Pareto Optimality

Quasi-linear preferences:-

$\max_{q \geq 0} \sum_{i=1}^{I} \phi_i(q) - c(q) \Rightarrow$ FONC $\sum_{i=1}^{I} \phi_i'(q^0) \leq c'(q^0)$ ('$=$' if $q^0 > 0$)

More generally, $\left(x_1^1, \ldots, x_L^1; \ldots; x_1^I, \ldots, x_L^I; q\right)$ satisfies

$$\sum_{i=1}^{I} \frac{\partial u^i\left(x^i, q^0\right)/\partial q}{\partial u^i\left(x^i, q^0\right)/\partial x_\ell^i} = MRT_{q\ell}.$$

Inefficiency of Private Provision of Public Goods

$$\max_{x_i \geq 0} \phi_i\left(x_i + \sum_{k \neq i} x_k^*\right) - p^* x_i$$

FONC $\phi_i'\left(x_i^* + \sum_{k \neq i} x_k^*\right) \leq p^*$ (with equality if $x_i^* > 0$)

Letting $x^* = \sum_{i=1}^{I} x_i^*$, we have

$\phi_i'(x^*) \leq p^*$ (with equality if $x_i^* > 0$)

Firm's supply:- q^* solves

$$\max_{q \geq 0} p^* q - c(q)$$

FONC $p^* \leq c'(q^*)$ with equality if $q^* > 0$

In equilibrium $q^* = x^*$.

$$\text{Letting } \delta_i = \begin{cases} 1 & \text{if } x_i^* > 0 \\ 0 & \text{if } x_i^* = 0 \end{cases}$$

FONCs of consumers' UMPs and Firm's PMP imply

$$\sum_{i=1}^{I} \delta_i \left[\phi_i'(q^*) - c'(q^*)\right] = 0.$$

Recalling $\phi_i' > 0$ and $c' > 0$, this implies that whenever $I > 1$ and $q^* > 0$ we have

$$\sum_{i=1}^{I} \phi_i'(q^*) > c'(q^*)$$

Solutions:

1. optimal direct provision:– govt chooses to produce q^0.
2. subsidize private provision.

E.g. Suppose $\phi_i(q) = \ln q$, and $c(q) = q^2/2$.

Efficient level given by solution to:

$$\max_q I \ln q - q^2/2 \Rightarrow \frac{I}{q^0} - q^0 = 0 \Rightarrow q^0 = I^{1/2}$$

Market solution: (x^*, p, q^*), where

1. Preference maximization: $x_i^* = x^*/I$ is solution to

$$\begin{aligned} \max_x \ln\left(x + \frac{[I-1]}{I}x^*\right) - p^*x \quad &\Rightarrow \quad \frac{1}{Ix} - p^* = 0 \\ &\Rightarrow \quad x^* = \frac{1}{p^*} \end{aligned}$$

2. Profit maximization: q^* solution to

$$\max_q p^*q - q^2/2 \Rightarrow p^* - q^* = 0 \Rightarrow q^* = p^*$$

3. Market-clearing: $x^* = q^* \Rightarrow p^* = 1 = x^* = q^*$

Lindahl Equilibrium – firm charges each consumer p_i^{**}

$$\max_{x_i \geq 0} \phi_i(x_i) - p_i^{**} x_i$$

FONC $x_i : \phi_i'(x_i^{**}) \leq p_i^{**}$ with equality if $x_i^{**} > 0$.

Firm solves

$$\max_{q \geq 0} \left(\sum_{i=1}^{I} p_i^{**} q \right) - c(q)$$

FONC $q^{**} : \sum_{i=1}^{I} p_i^{**} \leq c'(q^{**})$ with equality if $q^{**} > 0$

"Mkt-clearing" $x_i^{**} = q^{**}$ for all i.

$\sum_{i=1}^{I} \phi_i'(q^{**}) \leq c'(q^{**})$ with equality if $q^{**} > 0$

I.e. $q^{**} = q^0$.

For example above with $\phi_i(q) = \ln q$, and $c(q) = q^2/2$,
in Lindahl Equilibrium $p_i^{**} = I^{-1/2}$.

Multilateral Externalities

Depletable Externalities – [experience of externality by one agent reduces the amount that will be felt by other agents]

J firms generating externality $(\pi_j(.))$ and I consumers $(\phi_i(.))$

Assume $\pi_j^{'} > 0$, $\pi_j'' < 0$, $\phi_i'(.) < 0$ & $\phi''(.) < 0$

Firms: $h_j : \pi_j'(h_j^*) \leq 0$ with equality if $h_j^* > 0$.

Pareto optimal allocation: $\left(\tilde{h}_1^0, \ldots, \tilde{h}_I^0; h_1^0, \ldots, h_J^0\right)$ that solves

$$\max_{\left(\tilde{h}_1, \ldots, \tilde{h}_I; h_1, \ldots, h_J\right) \geq 0} \sum_{i=1}^{I} \phi_i\left(\tilde{h}_i\right) + \sum_{j=1}^{J} \pi_j(h_j) \text{ s.t. } \sum_{i=1}^{I} \tilde{h}_i = \sum_{j=1}^{J} h_j$$

Letting μ be multiplier on this constraint, FONC

$$\begin{aligned} \tilde{h}_i &: \phi_i'\left(\tilde{h}_i^0\right) \leq \mu \text{ with equality if } \tilde{h}_i^0 > 0,\ i = 1, \ldots, I \\ h_j &: \mu \leq -\pi_j'\left(h_j^0\right) \text{ with equality if } h_j^0 > 0,\ j = 1, \ldots, J. \end{aligned}$$

Non-depletable Externalities: $\tilde{h}_i = \sum_{j=1}^{J} h_j$, for all $i = 1, \ldots, I$

$$\max_{(h_1,\ldots,h_J)\geq 0} \sum_{i=1}^{I} \phi_i\left(\sum_{j=1}^{J} h_j\right) + \sum_{j=1}^{J} \pi_j\left(h_j\right)$$

$$\text{FONC } h_j : \sum_{i=1}^{I} \phi_i'\left(\sum_{j=1}^{J} h_j^0\right) \leq -\pi_j'\left(h_j^0\right) \text{ with equality if } h_j^0 > 0$$

- mkt-based solution would require "personalized" prices as in Lindahl equil.
- But given sufficient info. (i.e. work out optimal *agg.* level of externality) Govt can achieve optimality using quota.

Suppose $h^0 = \sum_{j=1}^{J} h_j^0$ permits issued, firm j receives $\bar{h}_j$ & $\sum_{j=1}^{J} \bar{h}_j = h^0$.

Each firm's demand for permits, h_j, solves

$$\max_{h_j \geq 0} \pi_j(h_j) + p_h^* \left(\bar{h}_j - h_j\right)$$

FONC $h_j : \pi_j'(h_j) \leq p_h^*$ with equality if $h_j > 0$.

Mkt-clearing: $\sum_{j=1}^{J} \left(\bar{h}_j - h_j\right) = 0$ & equil. price $p_h^* = -\sum_{i=1}^{I} \phi_i'\left(h^0\right)$

Private Information. Suppose $\phi(h, \eta)$ where $\eta \in \mathbb{R}$, is consumer's type, and $\pi(h, \theta)$ where $\theta \in \mathbb{R}$, is firm's type.

Decentralized Bargaining – suppose only 2 possible levels of externality 0 or 1.

Consumer makes 'take-it-or-leave-it' offer. Set,

$$\begin{aligned} b(\theta) &:= \pi(1, \theta) - \pi(0, \theta) > 0 \text{ measure of firm's benefit} \\ c(\eta) &:= \phi(0, \eta) - \phi(1, \eta) > 0 \text{ measure of consumer's cost} \end{aligned}$$

Denote by $G(b)$, CDF of b (density $g(b)$) & $F(c)$, CDF of c (density $f(c)$),

$$G(0) = F(0) = 0 \text{ and } G\left(\bar{b}\right) = F(\bar{c}) = 1.$$

Given consumer's cost $c > 0$, she chooses value of T to solve

$$\max_T [1 - G(T)][T - c]$$

$$\text{FONC } [1 - G(T)] - g(T)(T - c) = 0 \Rightarrow \frac{T - c}{T} = \frac{1 - G(T)}{g(T)T}$$

Solution has $T_c^* > c$.

No bargaining procedure can lead to efficient outcome for all values of b and c in this setting.

Groves-Clark Mechanism

Revelation Mechanism

Firm announces $\hat{b}$ & receives T_F from government, and
consumer announces $\hat{c}$ and receives T_C from government.

Where govt implements rule: allow pollution iff $\hat{b} > \hat{c}$ & transfers given by

$$T_F = \begin{cases} -\hat{c} & \text{if } \hat{b} > \hat{c} \\ 0 & \text{otherwise} \end{cases} \text{ and } T_c = \begin{cases} \hat{b} & \text{if } \hat{b} > \hat{c} \\ 0 & \text{otherwise} \end{cases}$$

*Weakly dominant for firm to announce $\hat{b} = b$
and for consumer to announce $\hat{c} = c$.*

Optimal amount of pollution but government runs deficit whenever $\hat{b} > \hat{c}$.