4 PARTIAL EQUILIBRIUM ANALYSIS

4.1 Perfectly Competitive Market Ref: MWG Chapter 10.C and 10.F (but also read 10.A &10.B)

Recall:

- consumers described by preferences over consumption bundles represented by utility functions

$$u^i : X^i \to \mathbb{R} \text{ where } X^i \subset \mathbb{R}^L_+$$

- producers described by production sets

$$Y^j \subset \mathbb{R}^L$$

  – divide goods into produced goods (outputs)

and factors of production (inputs)j

$$\begin{aligned} y^j &\in Y^j \\ y^j_\ell &> 0 \text{ output, call its price } p_\ell \\ y^j_\ell &< 0 \text{ input, call its price } \omega_\ell \end{aligned}$$

- In P.E. analysis of perfectly competitive market:-
  – focus on *one* produced good denoted by q

A. Consumers

1. happy to have good from any of the many producers/sellers
   - good undifferentiated and a commodity
2. consumers have perfect knowledge about prices charged by various sellers
3. consumers are price-takers.

From u^i derive $x^i\left(p, P, w^i\right)$

- assume prices of all other outputs (supplied goods) constant
- w^i wealth derived from ownership of firms and selling factors of production is fixed

Summing horizontally, obtain "market demand"

$$x(p) = \sum_{i=1}^{I} x^i\left(p, P, w^i\right)$$

B. Producers/Sellers

1. sellers happy to sell to any buyer.
2. perfect info. about competitors' prices
   - have ability to 'undercut' competitors
3. resale cannot be controlled
4. producers also price-takers (relative to ruling market price)

From SR prodn sets derive SR firm supply schedules

$$\max_{q_j} pq_j - c_j\left(q_j\right)$$

Define $q_j^{\min} = \underset{q>0}{\operatorname{argmin}}\, c_j(q_j)/q_j$.

FONC $\quad p \leq \max\left\{c_j'\left(q_j^*\right), c_j'\left(q_j^{\min}\right)\right\}$, (with equality if $q_j^* > \quad q_j^{\min}$)

i.e. $q_j(p) = \left(c_j'\right)^{-1}(p)$ if $p \geq c_j'\left(q_j^{\min}\right)$

Summing horizontally, obtain "market supply"

$$q(p) = \sum_{j=1}^{J} q_j(p)$$

4.1.1 Short-run Competitive Equilibrium

J fixed (because of adjustment and sunk costs etc.)

Short-run equil consists of $(q_1, \ldots, q_J)$ and $\left(x^1, \ldots, x^I\right)$ and price p s.t.

1. preference maximization: $x^i = x^i(p)$
2. profit maximization: q_j maximizes $pq_j - c_j(q_j)$,

   i.e.$p \leq \max\left\{c_j'\left(q_j^*\right), c_j'\left(q_j^{\min}\right)\right\}$, (with equality if $q_j^* > q_j^{\min}$)

3. market clearing: (supply = demand):

$$q(p) = \sum_{j=1}^{J} q_j(p) = \sum_{i=1}^{I} x_i(p) = x(p)$$

Special case: identical firms

Symmetric SR equil consists of $p, q, \left(x^1, \ldots, x^I\right)$ s.t.

1. preference max.: $x^i = x^i(p)$
2. profit max.: $q \in \underset{\widehat{q}}{\operatorname{argmax}}\ p\widehat{q} - c(\widehat{q})$
3. mkt clearing: $Jq = x(p)$

4.1.2 Long-run Competitive Equilibrium

Usual assumption – no factors fixed for firm (recall $0 \in Y$)

Free entry and exit.

Firms described by *LR* cost function $c_j(q)$, where $c_j(0) = 0$.

Again define $q_j^{\min} = \underset{q>0}{\operatorname{argmin}}\, c_j(q_j)/q_j$

LRCE consists of p, $(J,(q_1,\ldots,q_J))$ and $\left(x^1,\ldots,x^I\right)$ s.t.

1. Preference max: $x^i = x^i(p)$
2. Profit max.: q_j solves $\max_q pq - c_j(q)$
3. Market clearing

$$q(p) = \sum_{j=1}^{J} q_j(p) = \sum_{i=1}^{I} x_i(p) = x(p)$$

4. *free entry*:

$$\begin{aligned} & pq_1(p) - c_1(q_1(p)) \geq pq_2(p) - c_2(q_2(p)) \\ \geq\ & \ldots \\ \geq\ & pq_J(p) - c_J(q_J(p)) \\ \geq\ & 0 \geq pq_{J+1}(p) - c_{J+1}(q_{J+1}(p)) \end{aligned}$$

Special case: identical firms

Symmetric LRCE consists of $p, q, J, \left(x^1,\ldots,x^I\right)$ s.t.

1. preference max.: $x^i = x^i(p)$
2. profit max.: q solves $\max_{\widehat{q}} p\widehat{q} - c(\widehat{q})$
3. mkt clearing: $Jq = x(p)$
4. free entry $pq - c(q) = 0$

Short and Long Run Comparative Statics

Suppose initially in LR equil with J^0 firms with
$c(q) = K + \psi(q)$, $\psi(0) = 0$, $\psi'(0) > 0$, $\psi''(q) > 0$

Now suppose demand drops (permanently),
i.e. for demand $x(p;\theta)$, where $\frac{\partial}{\partial\theta}x(p,\theta) < 0$, consider increase in θ.

Short-run

1. π-max. condition
$$\frac{dp}{d\theta} = \psi''(q)\frac{dq}{d\theta}$$
2. Market clearing condition
$$J^0\frac{dq}{d\theta} = \frac{\partial}{\partial\theta}x(p,\theta) + \frac{\partial}{\partial p}x(p,\theta)\frac{dp}{d\theta}$$
Substituting yields:
$$J^0\frac{dq}{d\theta} = \frac{\partial}{\partial\theta}x(p,\theta) + \frac{\partial}{\partial p}x(p,\theta)\psi''(q)\frac{dq}{d\theta}$$
$$\Rightarrow \frac{dq}{d\theta} = \frac{\frac{\partial}{\partial\theta}x(p,\theta)}{J^0 - \frac{\partial}{\partial p}x(p,\theta)\psi''(q)} < 0$$
Hence
$$\frac{dp}{d\theta} = \psi''(q)\frac{dq}{d\theta} < 0$$

In *long-run* price adjusts to $c\left(q^0\right)/q^0$ as some firms exit industry and remaining firms expand output back to original optimal level (*minimum efficient scale*).

Two problems show up here:

1. J^0 and J^1 really need to be integers.

2. J needs to be "big". When J is small, firms should recognise effect their output decisions have on prices and strategic interaction of their competitors.

Resolution: model firms (or plant size) as literally *infinitesimally* small relative to market.

A continuum of potential firms can be achieved as the limit of letting the number of firms $\to \infty$ but keeping the market size finite.

4.2 Welfare in Partial Equilibrium (Ref: 10.E)

Inverse demand of consumers $p^i\left(x^i\right)$, aggregate inverse demand $P\left(x\right)$.

Consider differential change $\left(dx^1, \ldots, dx^I, dq_1, \ldots dq_J\right)$ satisfying $\sum_i dx^i = \sum_j dq_j$ and let dx denote $\sum_i dx^i$ and dq denote $\sum_j dq_j$.

Change in aggregate Marshallian surplus is then

$$dS = \sum_{i=1}^{I} p^i\left(x^i\right) dx^i - \sum_{j=1}^{J} c_j'\left(q_j\right) dq_j$$

Since $p^i\left(x^i\right) = P\left(x\right)$ for all i, $c_j'\left(q_j\right) = c'\left(q\right)$ for all j, and $dx = dq$, we get

$$dS = \left(P\left(x\right) - c'\left(x\right)\right) dx, \text{ so } S\left(x\right) = S_0 + \int_0^x \left(P\left(s\right) - c'\left(s\right)\right) ds$$

Alternatively, measure of welfare is CS + tax revenue + PS.

$$\int_{p_c}^{\infty} x\left(p\right) dp + \left(p_c - p_F\right) x\left(p\right) + \int_0^{p_F} q\left(p\right) dp$$

Consumers:

For each i, CS$^i = \int_{p_c}^{\infty} x^i\left(p\right) dp$, so CS $= \sum_i$CS^i

- assuming either income effects negligible or *no* income effects (e.g. quasi-linear utility).
- assuming all other markets operate efficiently, or no (compensated) cross-price effects

Producers:

PS is EV for producers. I.e. What is *equivalent* transfer to selling q at price p_F.

Hotelling's Lemma

$$\frac{\partial}{\partial p}\pi\left(p\right) = q\left(p\right)$$

- Implicit welfare function is sum of corresponding *money metric utility functions*
  - "dollar is a dollar" for given base prices

4.3 Monopoly (Ref: 12.B)

Single producer with cost function $c(q)$ who knows market demand $x(p)$.

$$\text{Pblm: } \max_{\langle p \rangle} \; px(p) - c(x(p))$$

or equivalently

$$\max_{\langle q \rangle} \; p(q)\, q - c(q) \; \text{ where } p(.) \text{ is inverse demand}$$

FONC

$$\begin{aligned} q \quad &: \quad q^m p'(q^m) + p(q^m) - c'(q^m) = 0 \\ &\Rightarrow \quad p(q^m) - c'(q^m) = -q^m p'(q^m) \\ &\Rightarrow \quad \frac{p - c'}{p} = -\frac{1}{\varepsilon_p} \; (\text{where } \varepsilon_p \equiv q'(p)\, p/q) \end{aligned}$$

Welfare

$$\begin{aligned} \text{DWL} &= \int_{q^m}^{q^c} (p(s) - c'(s))\, ds \\ &= -[\triangle CS \; + \; \triangle PS] - [p^m - c'(q^m)] \times q^m \\ &= \int_{p^c}^{p^m} x(p)\, dp + \int_{c'(q^m)}^{p^c} q(p)\, dp - [p^m - c'(q^m)] \times q^m \end{aligned}$$

Sufficient conditions for non-discriminating monopoly pricing.

1. Unit-demand by consumers & monopolist unable to discern any particular consumer's preferences
2. Multi-unit demand by consumers
   - monopolist unable to discern any particular consumer's preferences
   - resale of good is costless & monopolist cannot control (competitive) resale market.

   Denote 'price' for q units by $R(q)$.

   Since resale costless & resale mkt comp., unit price of good in resale mkt equals $\min_q\{R(q)/q\}$

   $\Rightarrow$ consumers will only buy qty $q^* = \operatorname{argmin}_q\{R(q)/q\}$
   and then resell each unit for $R(q^*)/q^*$

   Hence monopolist just as well off charging price per unit of $p(q^*) = R(q^*)/q^*$.

4.3.1 Price Discrimination

(a) First-Degree (or Perfect) Price Discrimination

Suppose consumer i has quasi-linear utility $u_i(q_i) + m_i$ [with $u_i(0) = 0$]

Monopolist makes "take-it-or-leave-it" offer (q_i, T_i) to each consumer.

Given offer (q_i, T_i), consumer i willing to accept offer iff $u_i(q_i) - T_i \geq 0$.

Consumer willing to pay $T_i = u_i(q_i)$, leaving her with *zero* surplus.

Monopolist's pblm becomes

$$\max_{\langle q_1,\ldots,q_I\rangle} \sum_{i=1}^{I} u_i(q_i) - c\left(\sum_{i=1}^{I} q_i\right)$$

Solution: FONC

$$q_i^* : u'(q_i^*) - c'\left(\sum_{j=1}^{I} q_j^*\right) \leq 0 \text{ (with " = " if } q_i^* > 0)$$

Solution maximizes aggregate surplus.

(b) Second-Degree Price Discrimination (Monopolistic Screening)
(ref: 14.C pp 488-501, particularly p500-501)

Suppose two types of consumers

λ : High marginal value $u_H(q) - T$, $u_H(0) = 0$;

$1 - \lambda$: Low marginal value $u_L(q) - T$, $u_L(0) = 0$.

$$u'_H(q) \geq u'_L(q) \text{, for all } q$$

Monopolist's pblm is to offer "menu" of $(qty, price)$ pairs that maximizes profits.

Assume monopolist has constant MC of production $= c$.

I. Type Observable: back to 1^{st}-degree price discrimination. Offer:

1. (q_H^*, T_H^*) to High MV customers;

2. (q_L^*, T_L^*) to Low MV customers; where,

$$\begin{aligned} u_H(q_H^*) - T_H^* &= 0 \text{ and } u'_H(q_H^*) = c \\ \& \; u_L(q_L^*) - T_L^* &= 0 \text{ and } u'_L(q_L^*) = c. \end{aligned}$$

II. Type Unobservable:

Menu must satisfy *self-selection* as well as *participation constraints.*

Formally pblm becomes:

$$\max_{\langle (q_H,T_H),(q_L,T_L)\rangle} \lambda T_H + (1-\lambda)\,T_L - (\lambda q_H + [1-\lambda]\,q_L)\,c$$

s.t. self-selection constraints

1. $u_H(q_H) - T_H \geq u_H(q_L) - T_L$
2. $u_L(q_L) - T_L \geq u_L(q_H) - T_H$

   and participation constraints
3. $u_H(q_H) - T_H \geq 0$
4. $u_L(q_L) - T_L \geq 0.$

Lemma 1. We can ignore (3)

Proof Whenever constraints (1) & (4) are satisfied, we have

$$u_H(q_H) - T_H \geq u_H(q_L) - T_L \geq u_L(q_L) - T_L \geq 0$$

[Since $u_H(0) = 0 = u_L(0)$ and $u'_H(\hat{q}) \geq u'_L(\hat{q})$, $\forall \hat{q} \Rightarrow u_H(q) \geq u_L(q)$]

Lemma 2. In an opitmal contract $u_L(q_L) - T_L = 0.$

Proof Suppose not, that is, there is an optimal solution

$$\left(\hat{q}_H, \hat{T}_H\right), \left(\hat{q}_L, \hat{T}_L\right) \text{ with } u_L(q_L) - T_L > \varepsilon > 0$$

Now consider an alternative menu in which T_H and T_L are both increased by ε. New contract still satisfies (4). (1) and (2) are still satisfied since ε subtracted from boht sides. But profit has been increased by ε, so original menu not optimal. A contradiction.

Lemma 3. In any optimal solution (i) $q_L \leq q_L^*$ and (ii) $q_H = q_H^*$
For proof refer to discussion and diagram in lecture.

Lemma 4. In any optimal solution $q_L < q_L^*$.
For proof refer to discussion and diagram in lecture.

Solution: $\left(\hat{q}_H, \hat{T}_H\right), \left(\hat{q}_L, \hat{T}_L\right)$

$$\begin{aligned} \text{where } \hat{q}_H &= q_H^* \text{ (i.e. } u_H'(\hat{q}_H) = c\text{)} \\ \hat{T}_H &= u_H(q_H^*) - \textstyle\int_{q=0}^{\hat{q}_L} \left[u_H'(q) - u_L'(q)\right] dq \\ \hat{T}_L &= u_L(\hat{q}_L) \end{aligned}$$

and where $\hat{q}_L$ satisfies

$$\lambda\left[u_H'(\hat{q}_L) - u_L'(\hat{q}_L)\right] = (1-\lambda)\left[u_L'(\hat{q}_L) - c\right]$$

$$\Rightarrow \quad \lambda u_H'(\hat{q}_L) + (1-\lambda)c = u_L'(\hat{q}_L)$$

- inefficiency of package designed for Low MV consumers (i.e. $\hat{q}_L < q_L^*$) designed to make $\left(\hat{q}_L, \hat{T}_L\right)$ *less attractive* for High MV consumers.

- In practice, monopolist often encourages self-selection not by adjusting quantities offered to consumers but by adjusting *quality* of good offered to different groups.

e.g. airlines

- $*$ unrestricted fare designed for business traveler v. restricted fare (incl. Saturday night stay) designed for non-business traveler.
- $*$ first-class travel v. coach.

(c) Third-Degree Price Discrimination
Monopolist charges different price per unit to (identifiably) different groups of consumers.

e.g. child & adult prices for movie tickets, senior citizens' discounts.

Compare π from serving only High MV consumers with π from serving both types.

Mathematically,
$$\max_{\langle q_H, q_L \rangle} p_H(q_H)\, q_H + p_L(q_L)\, q_L - c(q_H + q_L)$$

Optimal solution:

$$\begin{aligned} MR_H(q_H) &= c'(q_H + q_L) \\ MR_L(q_L) &\leq c'(q_H + q_L), \text{ with equality if } q_L > 0. \end{aligned}$$

4.4 Static Models of Oligopoly (Ref. 12.C)

Strategic interactions $\rightarrow$ Game Theory & Industrial Organisation

Basic problem – profit of firm not simply based on that firm's own actions *but* also depends on actions of competitors.

Postulate firms have *beliefs* about what other firms are doing.

Optimization now involves: choosing best action given belief about what other firms are doing.

Equil concept is *Nash equilibrium* (consistent and self-fulfilling beliefs)

Formally Nash equilibrium is profile of actions (strategies) $(s_1, \ldots, s_J)$ such that for each firm j, s_j is best/optimal response for firm j given its belief that other firms are undertaking

$$s_{-j} = (s_1, \ldots, s_{j-1}, s_{j+1}, \ldots, s_J).$$

4.4.1 Cournot (Quantity) Competition Model

Each firm simultaneously chooses quantity q_j

Price is determined to equate $S = D$.
That is, $p = p\left(\sum_j q_j\right)$ where $p(q)$ is inverse demand.

Define $b_j(q_{-j})$ as firm j's optimal quantity decision given belief other firms are choosing

$$q_{-j} = (q_1, \ldots, q_{j-1}, q_{j+1}, \ldots, q_J)$$

That is,

$$b_j(q_{-j}) = \underset{\langle q_j \rangle}{\text{argmax}}\ p\left(q_j + \sum_{k \neq j} q_k\right) q_j - c_j(q_j)$$

FONC

$$p'\left(q_j + \sum_{k \neq j} q_k\right) q_j + p\left(q_j + \sum_{k \neq j} q_k\right) - c_j'(q_j) = 0$$

(that is, view firm j acting as monopolist on *residual* demand curve.)

Hence a Nash equil strategy profile $(q_1^*, \ldots, q_J^*)$ satisfies for each j :

$$p'\left(\sum_{k=1}^{J} q_k^*\right) q_j^* + p\left(\sum_{k=1}^{J} q_k^*\right) - c_j'(q_j^*) = 0$$

Example: Say $p(q) = a - bq$, $c_j(q_j) = cq_j$.
Then there is exists a unique *symmetric* Nash equilibrium $(q^*, \ldots, q^*)$ where

$$p'(Jq^*)\, q^* + p(Jq^*) - c'(q^*) = 0$$

That is,

$$q^* = \frac{a - c}{(J+1)\, b}$$

And

$$Jq^* = \frac{J(a-c)}{(J+1)\, b} > \frac{a-c}{2b} = q^m$$

4.4.2 Betrand (Price) Competition Model

Take $J = 2$. Each firm simultaneously picks price p_j

$$x_j(p_j, p_k) = \begin{cases} x(p_j) & \text{if } p_j < p_k \\ x(p_j)/2 & \text{if } p_j = p_k \\ 0 & \text{if } p_j > p_k \end{cases}$$

If $c_1(q) = c_2(q) = cq$ then the *unique* Nash equil is $p_1 = p_2 = c$.

4.4.3 Capacity Constraints

Often natural to suppose firms operate under conditions of eventual DRS (especially in SR when some factors, e.g. capital stock, are fixed).

With capital constraints, no longer sensible to assume price announcement is commitment to provide *any* demanded quantity since costs of order larger than capacity are infinite (or at least prohibitively large).

So assume price announcements are commitments to supply up to capacity.

Assume capacities commonly known among firms. Consider following example with

$$c(q) = \begin{cases} K + \psi q & q \leq \bar{q} \\ \infty & q > \bar{q} \end{cases}$$

where $\bar{q} = 3x(\psi)/4$.

Notice $p_1^* = p_2^* = \psi$ no longer an equilibrium.

More generally, whenever $\bar{q} < x(\psi)$, each firm can *assure* itself strictly positive level of sales and strictly positive operating profits by setting its price below $p(\bar{q})$ but above ψ.

Consider following two-stage game

1. Each firm simultaneously sets its capacity.

2. Given capacity choices, firms simultaneously set prices.

Solving game backwards:- i.e. requiring price choice to be N.E. *given* capacity choices, the *unique subgame perfect* Nash equilibrium involves "Cournot outcome" in capacity choices (Kreps-Scheinkman, 1983).

That is, may view Cournot quantity competition as capturing LR competition through capacity choice, with price competition occurring in SR given chosen levels of capacity.

4.4.4 Product Differentiation

Firm j faces *continuous* demand function $x_j\left(p_j, p_{-j}\right)$.

Solve:

$$\max_{p_j} \left(p_j - c\right) x_j\left(p_j, \bar{p}_{-j}\right)$$

where $\bar{p}_{-j}$ is firm j's expectation about rivals' price choices.

$\bar{p}$ is a Nash equilibrium, if for all j

$$\bar{p} \in \operatorname*{argmax}_{p_j} \left(p_j - c\right) x_j\left(p_j, \bar{p}_{-j}\right)$$

Spatial Models of Production Differentiation.

Example: Linear City Model of Product Differentiation.

Mass 1 of consumers uniformly distributed along unit interval.

Consumer's location indexed by $z \in [0,1]$

c is constant MC of production for both firms.

Total cost of buying from firm j for consumer located at distance d from j is $p_j + td$, which yields net utility $v - p_j - td$.

Given p_1 and p_2, $\hat{z}$ is consumer who is indifferent between buying from either firm, i.e.,

$$p_1 + t\hat{z} = p_2 + t(1-\hat{z}) \Rightarrow \hat{z} = \frac{t + p_2 - p_1}{2t}$$

Lemma. If $v > c + 2t$ then firms never want to set prices at level that causes some consumers not to purchase from either firm.

Proof. Suppose to contrary, in situation where for each $z \in (\hat{z}_1, \hat{z}_2)$,

$$\min(p_1 + tz, p_2 + t(1-z)) > v$$

And

$$\begin{aligned} v - p_1 - tz &\geq 0 \text{ for all } z \in [0, \hat{z}_1] \\ v - p_2 - t(1-z) &\geq 0 \text{ for all } z \in [\hat{z}_2, 1]. \end{aligned}$$

Consider a small reduction in firm 1's price p_1. As long as firm 1 does not start to compete with firm 2, its sales are given by consumer at location x_1 who is indifferent between buying from firm 1 and not buying at all:

$$v - p_1 - tx_1 = 0 \Rightarrow \text{Demand } x_1(p_1) = \frac{(v - p_1)}{t}$$

So

$$\pi_1(p_1) = (p_1 - c)\, x_1(p_1) = \frac{(p_1 - c)(v - p_1)}{t}.$$

Differentiating wrt p_1,

$$\pi_1'(p_1) = \frac{(v + c - 2p_1)}{t}.$$

By assumption, some consumers strictly prefer not to buy rather than buy from firm 1. This is true for consumers at location $z = 1$, which means $p_1 > v - t$. Hence

$$\pi_1'(p_1) < \frac{(c - v + 2t)}{t} < 0, \text{ by assumption.}$$

Therefore, by lowering p_1 slightly, firm 1 could raise its profit, which contradicts assumption firm 1 is playing its best response.

So

$$x_1(p_1, p_2) = \begin{cases} 0 & \text{if } p_1 > p_2 + t \\ (t + p_2 - p_1)/2t & \text{if } p_1 \in [p_2 - t, p_2 + t] \\ 1 & \text{if } p_1 < p_2 - t \end{cases}$$

Similarly,

$$x_2(p_1, p_2) = \begin{cases} 0 & \text{if } p_2 > p_1 + t \\ (t + p_1 - p_2)/2t & \text{if } p_2 \in [p_1 - t, p_1 + t] \\ 1 & \text{if } p_2 < p_1 - t \end{cases}$$

So firm j's pblm, given firm k is setting price $\bar{p}_k$:-

$$\max_{p_j \in [\bar{p}_k - t, \bar{p}_k + t]} \frac{(p_j - c)(t + \bar{p}_k - p_j)}{2t}$$

FONC

$$t + \bar{p}_k + c - 2p_j \begin{cases} \leq 0 & \text{if} \quad p_j = \bar{p}_k \\ = 0 & \text{if} \quad p_j \in (\bar{p}_k - t, \bar{p}_k + t) \\ \geq 0 & \text{if} \quad p_j = \bar{p}_k + t \end{cases}$$

Hence, the best response function is given by

$$b_j(\bar{p}_k) = \begin{cases} \bar{p}_k + t & \text{if} \quad \bar{p}_k \leq c - t \\ (t + \bar{p}_k + c)/2 & \text{if} \quad \bar{p}_k \in (c - t, c + 3t) \\ \bar{p}_k - t & \text{if} \quad \bar{p}_k \geq c + 3t \end{cases}$$

Symmetric solution $p_1^* = p_1^* = p^*$, i.e.,

$$\begin{aligned} & p^* = b(p^*) \Rightarrow p^* = (t + p^* + c)/2 \\ \Rightarrow \quad & p^* = c + t. \end{aligned}$$