COMPETITIVE MARKETS

(Preview for Next Semester)

- Elements:

$$\begin{aligned} I \text{ consumers } i &= 1,\ldots,I \\ J \text{ firms } j &= 1,\ldots,J \\ L \text{ goods } \ell &= 1,\ldots,L \end{aligned}$$

$$u^i : X^i \to \mathbb{R} \text{ where } X^i \subset \mathbb{R}^L_+$$

- $\succsim_i \subset X^i \times X^i$, where $X^i \subset \mathbb{R}^L$, represented by utility fn $u^i(\cdot)$
- $\omega = (\omega_1,\ldots,\omega_L) \geq 0$, economy-wide *endowment*
- $Y^j \subset \mathbb{R}^L$, production set for firm j.

If $(y^1,\ldots,y^J) \in Y^1 \times \ldots \times Y^J$ then total (net) amount of good ℓ available to the economy is $\omega_\ell + \sum_j y^j_\ell$.

DEFN: An *economic allocation* $(x^1,\ldots,x^I; y^1,\ldots,y^J)$ is a specification of a consumption vector $x^i \in X^i$ for each consumer $i = 1,\ldots,I$ and a production vector $y^j \in Y^j$ for each firm $j = 1,\ldots,J$. The allocation is *feasible* if

$$\sum_{i=1}^{I} x^i_\ell \leq \omega_\ell + \sum_j y^j_\ell, \text{ for } \ell = 1,\ldots,L.$$

Pareto Optimality A feasible allocation $(x^1,\ldots,x^I; y^1,\ldots,y^J)$ is *Pareto optimal* (or *Pareto efficient*) if there is no other feasible allocation $(\hat{x}^1,\ldots,\hat{x}^I; \hat{y}^1,\ldots,\hat{y}^J)$ such that $u^i(\hat{x}^i) \geq u^i(x^i)$ for all $i = 1,\ldots,I$ and $u^i(\hat{x}^i) > u^i(x^i)$ for some i.

Competitive Equilibria in Private Ownership Economy

To above description of the economy specify:

1. initial allocation of endowments

$$\omega^i = \left(\omega_1^i, \ldots, \omega_L^i\right)$$
$$\text{where } \sum_{i=1}^{I} \omega_\ell^i = \omega_\ell \text{ for } \ell = 1, \ldots, L.$$

2. consumer i's claims to profits of firms:- θ^{ij} is consumer i's share of profits accruing to firm j, where

$$\text{where } \sum_{i=1}^{I} \theta^{ij} = 1 \text{ for } j = 1, \ldots, J.$$

DEFN: The allocation $\left(x^1, \ldots, x^I; y^1, \ldots, y^J\right)$ and price vector $p \in \mathbb{R}_{++}^L$ constitutes a *competitive* (or *Walrasian*) equilibrium if the following conditions are satisfied:-

(i) Profit Maximization: For each firm j, y^j solves

$$\max_{\hat{y}^j \in Y^j} p.y^j$$

(ii) Utility Maximization: For each consumer i, x^i solves

$$\max_{\hat{x}^i \in X^i} u^i\left(x^i\right) \text{ s.t. } p.x^i \leq p.\omega^i + \sum_{j=1}^{J} \theta^{ij}\left(p.y^j\right).$$

(iii) Market Clearing: For each good $\ell = 1, \ldots, L$

$$\sum_{i=1}^{I} x_\ell^i = \omega_\ell + \sum_j y_\ell^j.$$

Walras' Law

If the allocation $\left(x^1, \ldots, x^I; y^1, \ldots, y^J\right)$ and price vector $p \gg 0$ satisfy market clearing for all goods $\ell \neq k$ and if every consumer's budget constraint is satisfied with equality, so that $p.x^i \leq p.\omega^i + \sum_{j=1}^J \theta^{ij}\left(p.y^j\right)$ for all i, then the market for good k also clears.

Proof: Adding up the consumer's budget constraints over the I consumers and rearranging terms, yields

$$\sum_{\ell \neq k} p_\ell \left(\sum_{i=1}^I x_\ell^i - \omega_\ell - \sum_j y_\ell^j\right) = -p_k \left(\sum_{i=1}^I x_k^i - \omega_k + \sum_j y_k^j\right)$$

By market clearing in all goods $\ell \neq k$, LHS of this equation is equal to zero. Thus RHS must be equal to zero as well. Since $p_k > 0$ we have market clearing in good k, as desired. ■

First Fundamental Theorem of Welfare Economics

If preferences are locally non-satiated, and if (x, y, p) is a Walrasian equilibrium, then the allocation (x, y) is Pareto optimal.

Proof: Consider another allocation $(\hat{x}, \hat{y})$ that Pareto dominates (x, y). That is, $u^i\left(\hat{x}^i\right) \geq u^i\left(x^i\right)$ for all $i = 1, \ldots, I$, with *strict* inequality for at least one individual.

By utility maximization,

$$u^i\left(\hat{x}^i\right) > u^i\left(x^i\right) \Rightarrow p.\hat{x}^i > p.x^i = p.\omega^i + \sum_{j=1}^J \theta^{ij}\left(p.y^j\right)$$

Futhermore, by local non-satiation

$$u^i\left(\hat{x}^i\right) \geq u^i\left(x^i\right) \Rightarrow p.\hat{x}^i \geq p.x^i = p.\omega^i + \sum_{j=1}^J \theta^{ij}\left(p.y^j\right)$$

(To see why, suppose not, that is suppose $p.\hat{x}^i < p.x^i$ for some i, then by ℓ*ns*, $\exists\ \tilde{x}^i$ s.t. $p.\tilde{x}^i < p.x^i$ and $u^i\left(\tilde{x}^i\right) > u^i\left(\hat{x}^i\right) \geq u^i\left(x^i\right)$, contradicting that x^i was utility maximizing.)

Hence we have

$$\sum_{i=1}^{I} p.\hat{x}^i > \sum_{i=1}^{I} p.x^i = p.\omega + \sum_j p.y^j.$$

Moreover, because y^j is profit-maximizing for firm j at prices p we have

$$p.\omega + \sum_j p.y^j \geq p.\omega + \sum_j p.\hat{y}^j$$

Thus,

$$\sum_{i=1}^{I} p.\hat{x}^i > p.\omega + \sum_j p.\hat{y}^j.$$

But then $(\hat{x}, \hat{y})$ cannot be feasible since

$$\sum_{i=1}^{I} \hat{x}^i = \omega + \sum_j \hat{y}^j \Rightarrow \sum_{i=1}^{I} p.\hat{x}^i = p.\omega + \sum_j p.\hat{y}^j.$$

FFWT Proof (in words)

At any feasible allocation $(\tilde{x}, \tilde{y})$, total cost of consumption bundles $\left(\tilde{x}^1, \ldots, \tilde{x}^I\right)$ evaluated at prices p, must be equal to social wealth evaluated at those prices, namely

$$p.\omega + \sum_{j=1}^{J} p.\tilde{y}^j.$$

Now, since preferences are ℓ*ns* the allocation $(\hat{x}, \hat{y})$ Pareto dominates (x, y) then total cost of consumption bundles $\left(\hat{x}^1, \ldots, \hat{x}^I\right)$ must exceed total cost of equilibrium consumption allocation, i.e.,

$$p.\left(\sum_{i=1}^{I} \hat{x}^i\right) > p.\left(\sum_{i=1}^{I} x^i\right) = p.\omega + \sum_{j=1}^{J} p.y^j$$

Since by market clearing:

$$\sum_{i=1}^{I} x^i = \omega + \sum_{j=1}^{J} y^j$$

But by profit maximization, there are no technically feasible production plans that attain a value of social wealth at prices in excess of $p.\omega + \sum_{j=1}^{J} p.y^j$.