

Rice University

Mid-Semester Examination Fall 2005

ECON 501: Advanced Microeconomic Theory

Duration: One Hundred (100) minutes

Permitted Materials: English/Foreign Language Dictionaries and non-programmable calculators

You should answer all questions in Part A and all parts of question 4 in Part B. All questions in Part A are of equal weight, and Part A accounts for half of the total 100 points.

Part A

1. Consider the following function.

$$V(p_1, p_2, w) = \frac{W}{p_1} - \frac{p_2}{W}$$

State the properties an indirect utility must satisfy and show that the function above satisfies all of them. Derive the uncompensated demand.

2. Consider the following indirect utility function.

$$V(p_1, p_2, w) = \left(\frac{-p_1}{W} \right) + \left(\frac{-p_2}{W} \right)$$

*Derive the **compensated** demand and the substitution matrix. State the properties a substitution matrix must satisfy and show that these properties all hold for the substitution matrix you have computed.*

3. *State and prove the compensated law of demand for a world with $L > 1$ commodities and draw a diagram illustrating its implication in a two-commodity world.*

Part B

4. “**Necessities: You can’t live by bread alone?**” Mr Dubois consumes just ‘food’ and wine. Let y be the quantity of ‘food’ he consumes; let x_3 be quantity of wine he consumes; and let w denote his wealth. His preferences between ‘food’ and wine are given by

$$U(y, x_3) = \alpha \ln y + (1 - \alpha) \ln x_3.$$

where $\alpha \in (0, 1)$. Mr Dubois can buy wine but he cannot buy ‘food’ as such. Rather, he has to buy two basic goods, bread and cheese, and use these as inputs to produce ‘food’. His sub-utility function for ‘food’ is given by: $y = \min\{x_1, 2x_2 - 2\}$ where x_1 is the quantity of cheese, and x_2 is the quantity of bread.

- (a) [5 points] Write down the form of Mr Dubois’ overall utility function $u(x_1, x_2, x_3)$ representing his preferences for the three basic goods.
- (b) [10 points] Consider Mr Dubois’ production of food and his expenditure on food. Show that Mr Dubois’ food expenditure function is given by

$$e(p_1, p_2, y) = \left(p_1 + \frac{p_2}{2}\right) y + p_2$$

where p_1 is the price of cheese, and p_2 is the price of bread.

- (c) [10 points] State Shephard’s lemma. Verify that this expenditure function satisfies Shephard’s lemma and all of the properties an expenditure function is required to satisfy.
- (d) [5 points] Consider Mr Smith choosing quantities y of ‘food’ and x_3 of wine to maximize his overall utility. Write down the budget constraint for this problem.
- (e) [10 points] Derive Mr Smith’s **uncompensated** demand for the basic goods: cheese, bread and wine. Use your answer to show whether or not these preferences are homothetic.
- (f) [10 points] For Cobb-Douglas preferences, we are used to finding that the expenditure shares $p_j x_j / w$ are constant, and hence that consumption of good j does not depend on prices other than p_j . Give an intuition why this is not the case here for any of the three basic goods (cheese, bread and wine). How could we adapt the notion of an expenditure share appropriately for these preferences so that the adapted expenditure share of wine is constant? Interpret your answer in terms of ‘disposable wealth’.