Rice University

Fall Semester Final Examination 2007

ECON501 Advanced Microeconomic Theory

Writing Period: **Three Hours**

Permitted Materials: English/Foreign Language Dictionaries and non-programmable calculators

There are five pages (including this title page) and five questions. You should attempt all parts of all five questions. The total points for the exam is one hundred and eighty (180).

1. **[60 Points]**

Consider a preference relation that can be represented by the utility fuction

$$u(x_1, x_2, x_3, x_4) = \left(\frac{x_1}{\alpha}\right)^{\alpha} \left(\frac{x_2}{1-\alpha}\right)^{1-\alpha} + 2\sqrt{\left(\frac{x_3}{\beta}\right)^{\beta} \left(\frac{x_4}{1-\beta}\right)^{1-\beta}}.$$

(a) Derive the uncompensated demand function for the region of the parameter space in which the consumer is choosing positive amounts of all four commodities. Derive the marginal propensities to expend on each commodity, i.e. find $p_\ell(\partial x_\ell/\partial w)$. **[Hint: Notice**

$$u(x_1, x_2, x_3, x_4) = u_A(x_1, x_2) + u_B(x_3, x_4)$$

Consider the two-stage budgeting problem in which the individual divides his wealth between expenditure on group A commodities and expenditure on group B commodities, and then allocates the expenditure on group A (respectively, on group B) in a group-utility maximizing way.] (20 points).

(b) Derive the indirect utility function. (5 points).

(c) Derive the compensated demand function. (5 points).

Recall in lectures that the equivalent variation of a change in prices and income from (p^0, w^0) to (p^1, w^1) was defined to be

$$\text{EV} = e\left(p^0, v\left(p^1, w^1\right)\right) - w^0.$$

Further recall that if **wealth is unaltered** and the change in prices is caused by the imposition of commodity taxes then the deadweight loss (DWL) or excess burden of the taxes is given by

$$\text{DWL} = -\,\text{EV} - \sum_{\ell=1}^{L} t_\ell x_\ell\left(p^1, w^0\right).$$

(d) Briefly explain why this measure may be viewed as a deadweight loss to (social) economic efficiency. (5 points).

(e) Suppose the initial budget constraint has $p^0 = (1,1,1,1)$ and $w^0 = 36$. Suppose further that the consumer's preferences parameters are $\alpha = 1/2$ and $\beta = 1/3$. Calculate the revenue raised and the DWL of the imposition of a specific tax of 3 on good 1, that leads to the price of good 1 rising to 4. (10 points).

(f) Again starting from the initial position corresponding to $p^0 = (1,1,1,1)$ and $w^0 = 36$ compute the revenue raised and the DWL arising from the imposition of a specific tax of 3 on good 1, and specific taxes of 1 on both goods 3 and 4 resulting in the price of good 1 rising to 4 and the prices of goods 3 and 4 both rising to 2. (10 points).

(g) You should have found the DWL computed in **(e)** was *greater* than the DWL computed in **(f)**. Explain the underlying economic intuition for why imposing additional 'distortionary' taxes on goods 3 and 4 in part (e) not only raises more revenue but can actually *reduce* the DWL that arose from imposing a distortionary tax just on good 1 (that is, induce an overall *increase* in economic efficiency.) (5 points).

2. **[15 Points]** Consider the following four lotteries:

$$L_1(x) = \begin{cases} 1 & \text{if } x = \$3,000 \\ 0 & \text{otherwise} \end{cases}, \quad L_2 = \begin{cases} 0.8 & \text{if } x = \$4,000 \\ 0.2 & \text{if } x = \$0 \\ 0 & \text{otherwise} \end{cases}$$

$$L_3 = \begin{cases} 0.25 & \text{if } x = \$3,000 \\ 0.75 & \text{if } x = \$0 \\ 0 & \text{otherwise} \end{cases} \quad \text{and } L_4 = \begin{cases} 0.2 & \text{if } x = \$4,000 \\ 0.8 & \text{if } x = \$0 \\ 0 & \text{otherwise} \end{cases}$$

Show that if for the preference relation $\succcurlyeq$ we have

$$L_1 \succ L_2 \text{ and } L_4 \succ L_3$$

then $\succcurlyeq$ does not admit an expected utility representation. Which of the axioms for subjective expected utility theory does this pattern of preferences violate? State this axiom and show how the this pattern of preference is inconsistent with the axiom.

3. **[15 Points]** An individual is an expected utility maximizer described by the intertemporally additive preference-scaling function

$$u(c_0) + \beta u(c_1)$$

where $u(\cdot)$ is a strictly concave function with $u'''(c) > 0$. The individual has current income I_0. The individual can buy bonds at unit price $p = \beta$ which pay out in the next period one unit of consumption per unit of bond held. Compare the individual's demand for bonds in the case where her future income is certain and equal to I_0, and the situation in which there is a fifty percent chance future income is $I_0 + \varepsilon$ and a fifty percent chance future income is $I_0 - \varepsilon$.

4. **[30 Points]** A price-taking firm produces output q from inputs z_1 and z_2 according to a differentiable concave production function $f(z_1, z_2)$. The price of its output is $p > 0$ and the price of its inputs are $(w_1, w_2) \gg 0$. However, there are two unusual things about this firm. First, rather than maximizing profit, the firm maximizes revenue (the manager wants his firm to have bigger dollar sales than any other). Second, the firm is cash constrained. In particular, it has only C dollars on hand before production and, as a result, its total expenditures on inputs cannot exceed C.

Suppose one of your econometrician colleagues tells you that she has used repeated observations of the firm's revenue under various output prices, and levels of the financial constraint and has determined that the *log* of the firm's revenue level R can be expressed as the following function of the variables (p, w_1, w_2, C) :

$$\ln R(p, w_1, w_2, C) = \ln p + [\gamma + \ln C - \alpha \ln w_1 - (1-\alpha)\ln w_2]$$

(a) What is the firm's use of inputs z_1 and z_2 and its output q when prices are (p_1, w_1, w_2) and it has C dollars of cash on hand? (15 points).

(b) Using your answer to (a) show the firm's production function is Cobb-Douglas. (15 points).

5. [**15 Points**] A real-valued function $f : \mathbb{R}^n \to \mathbb{R}$, is called **superadditive** if $f(x^1 + x^2) \geq f(x^1) + f(x^2)$. Show that every cost function is superadditive in input prices. Use this to prove the cost function is non-decreasing in input prices without requiring it to be differentiable.

6. [**45 Points**] **Throughout this question, assume all functions mentioned are twice continuously differentiable.**

In Ruritania, the electricity industry is perfectly competitive both in the input and output markets. Let $B_e(q)$ represent a schedule of benefits generated by the consumption of electricity, with $B_e(0) = 0$, $B_e'(q) > 0$ and $B_e''(q) < 0$. Production opportunities in this industry is summarized by the aggregate production function:

$$q = f(K_e, L, z),$$

where q is the (maximum) amount of electricity that can be generated when K_e units of capital are employed in electricity generation, in conjunction with L units of labor and z units of coal. Suppose f is strictly increasing in all its arguments and strictly concave and that capital and labor are elastically supplied at prices r and w, respectively. Suppose further that coal is competitively supplied according to a technology embodied in the cost function $C(m)$ (that is, the cost of supplying m units of coal to the market is $C(m)$, where $C(0) = 0$, $C' > 0$ and $C'' > 0$.)

Using coal to produce electricity generates as sulphur as a by-product. Left untreated sulphur is a pollutant but it can be made valuable if delivered in a pure state. Suppose the by-production of sulphur from burning coal is a linear activity, so that without loss of generality we can choose units for sulphur to be such that using z units of coal generates z units of sulphur as by-product. Let

$$y = g(K_s, z)$$

represent the production of pure sulphur from this joint product with K_s being the units of capital involved in the "purification" process (also available at [constant] unit price of r). The function g is strictly increasing in all arguments, strictly concave, and

$$\lim_{K_s \to \infty} g(K_s, z) \leq z \text{ for all } z.$$

The benefit for purified sulphur is given by the schedule $B_s(y)$, with $B_s'(y) > 0$, $B_s''(y) < 0$ and $B_s(0) = 0$. Unpurified sulphur will be emitted as a pollutant into the environment. The amount of pollutant d will be

$$d = z - g(K_s, z).$$

The social harm it will cause is given by the schedule $H(d)$, with $H' > 0$, $H'' > 0$ and $H(0) = 0$.

(a) Characterize the economic efficient outcome for this configuration. (10 points).

(b) Show that efficiency can be achieved even if $H(d)$ were ignored by producers in a competitive market if the government imposed an appropriate emission charge on d. (15 points).

(c) Show that a system that charged a tax t on the use of coal and paid a subsidy s for the production of purified sulphur could also achieve optimality. (15 points).

(d) Contrast the informational requirements of these alternative systems (that is, the regulative structures embodied in parts (*b*) and (*c*).) (5 points).