Rice University

Fall Semester Final Examination 2006

ECON501 Advanced Microeconomic Theory

Writing Period: **Three Hours**

Permitted Materials: English/Foreign Language Dictionaries and non-programmable calculators

There are five pages (including this title page) and five questions. You should attempt all parts of all five questions. The total points for the exam is one hundred and eighty (180).

1. **[90 Points]**

(a) Solve the utility maximization problem and derive the indirect utility function for preferences represented by the utility function

$$u(x_1, x_2) = \ln x_1 + x_2$$

Verify the indirect utility function satisfies all the requisite properties. (25 points).

Consider an economy with a continuum of consumers of total measure 1. Half of the consumers are of type I whose preferences can be represented by the utility function

$$u^I(x_1, x_2) = \ln x_1 + x_2.$$

The remaining half of the consumers are of type II whose preferences may be represented by the utility function

$$u^{II}(x_1, x_2) = x_1 + \ln x_2.$$

Suppose every consumer has the same wealth equal to the per-capita wealth $\bar{W}$.

(b) Show that the *per capita* demand for goods 1 and 2, can be expressed as the following function of p_1, p_2 and per capita wealth $\bar{W}$:

$$\begin{aligned}
\bar{x}_1\left(p_1, p_2, \bar{W}\right) &= \frac{\min\left(p_2, \bar{W}\right) + \max\left(\bar{W} - p_1, 0\right)}{2p_1} \\
\bar{x}_2\left(p_1, p_2, \bar{W}\right) &= \frac{\max\left(\bar{W} - p_2, 0\right) + \min\left(p_1, \bar{W}\right)}{2p_1}
\end{aligned}$$

Compute the per capita demand and the demands for individuals of type I and type II for goods 1 and 2 when $p_1 = p_2 = 1$ and per capita wealth $\bar{W} = 3/2$. (10 points).

For the remainder of the question assume all individuals have the same wealth, that is, they all have wealth equal to the per capita wealth $\bar{W}$, and further assume $\bar{W} > \max(p_1, p_2)$.

(c) When $\bar{W} > \max(p_1, p_2)$, show that the per capita demand you derived in part (b) is the solution to the following utility maximization problem of a representative consumer with preferences represented by the *indirect* utility function

$$\bar{V}\left(p_1, p_2, \bar{W}\right) = \frac{\bar{W} + p_1 + p_2}{\sqrt{p_1 p_2}}$$

(15 points).

(d) Derive the substitution matrices for consumers of type I, for consumers of type II and for the representative consumer. Denote these matrices by $S^{I}\left(p_1, p_2, W^{I}\right)$ $S^{II}\left(p_1, p_2, W^{II}\right)$ and $\bar{S}\left(p_1, p_2, \bar{W}\right)$, respectively. Evaluate these matrices for $p_1 = p_2 = 1$ and $W^{I} = W^{II} = \bar{W} = 3/2$. Show that

$$C\left(1,1,3/2\right) = \frac{1}{2}S^{I}\left(1,1,3/2\right) + \frac{1}{2}S^{II}\left(1,1,3/2\right) - \bar{S}\left(1,1,3/2\right)$$

is negative definite. Explain the welfare significance of this. (40 points).

2. **[30 Points]** An individual taxpayer has an income y that he should report to the tax authority. Tax is payable at a constant proportionate rate t. The taxpayer reports x where $0 \leq x \leq y$ and is aware that the tax authority audits some tax returns. Assume that the probability that the taxpayer's report is audited is π, that when an audit is carried out the true taxable income becomes public knowledge and that, if $x < y$, the taxpayer must pay both the underpaid tax and a surcharge of s times the underpaid tax.

(a) If the taxpayer chooses $x < y$, show that disposable income c in the two mutually exclusive events NA (taxpayer is not audited) and A (taxpayer is audited) is given by

$$\begin{aligned} c_{NA} &= y - tx \\ c_A &= (1 - t - s)\,y + stx \end{aligned}$$

(5 points).

Assume that the individual is an expected utility maximizer with a preference scaling function over consumption of $u(.)$, where u is increasing and strictly concave.

(b) Write down the first order necessary condition for an interior maximum. Explain why or why not this condition is sufficient for an interior maximum. (5 points).

(c) Show that if $1 - \pi - \pi s > 0$ then the individual will definitely under-report income. (5 points).

Assume for the rest of the question that the optimal report x^* satisfies $0 < x^* < y$.

(d) Show that if the surcharge is raised then under-reported income will decrease. (5 points).

(e) If true income increases will under-reported income increase or decrease? Briefly explain the reason for your answer? [Hint: What property of the preference scaling function will this 'wealth' effect depend upon?] (10 points).

3. **[15 Points]** For any homothetic production function show that the cost function must be expressible in the form

$$c(w,q) = c(w,1)\, h(q),$$

where $h(.)$ is an increasing function and $c(w,1)$ is concave in w.

4. **[15 Points]** Consider an economy with a fixed number of firms, each characterized by its production set. Suppose the standard assumptions hold. In particular, suppose there are no production externalities. That is, the production possibilities available to one firm are unaffected by the production plan adopted by any other firm. Suppose all firms are price-takers. Let y^0 denote the *aggregate* supply associated with prices p^0 and let y^1 denote the *aggregate* supply associated with prices p^1. Assuming all firms are profit-maximizers state and prove the relationship that must hold between (p^0, y^0) and (p^1, y^1).

5. **[30 Points]** A government owned enterprise (GOE) generates electricity for the town of Wagga with a constant returns to scale technology with constant marginal cost of electricity generation equal to c. There are two-types of households who demand electricity in Wagga. The fraction λ are H-types, while the remaining fraction $(1-\lambda)$ are L -types. If an H-type (respectively, L-type) household consumed q units of electricity and paid T in total then the consumer surplus enjoyed by that household is given by

$$\begin{aligned} CS_H(q,T) &= u_H(q) - T \\ (\text{respectively, } CS_L(q,T) &= u_L(q) - T), \end{aligned}$$

where u_H and u_L are both increasing, twice continuously differentiable and strictly concave functions with $u_H(0) = u_L(0) = 0$, $u'_H(0) = \alpha$, $u'_H(\bar{q}) = 0$, and $u'_H(q) > u'_L(q)$ for all $q \in [0, \bar{q})$.

For this question assume that the GOE cannot distinguish H type households from L type households.

(a) Design a pricing scheme that maximizes the sum of consumer and producer surplus. (7 points).

Now suppose the government faces an excess burden in raising revenue from the imposition of distorting taxes in other markets.

(b) If the marginal excess burden of raising a dollar of revenue is $\gamma > 0$, explain why the *social opportunity cost* to the government of forgoing a dollar in profit from electricity generation is $1 + \gamma$. (3 points).

(c) Design a non-linear pricing scheme that maximizes

$$\lambda CS_H + (1-\lambda) CS_L + (1+\gamma)\pi$$

where, recall, CS_H (respectively, CS_L) is the consumer surplus enjoyed by an H-type (respectively, L-type) household and π is the *per-household* profit earned by the GOE.

[Hint: Find the 'optimal' two-element "menu" $\left(\hat{q}_H, \hat{T}_H\right)$ and $\left(\hat{q}_L, \hat{T}_L\right)$, where the first package is designed for H type households and the second package is designed for L type households. You should be able to show (diagrammatically) that $u_H'(\hat{q}_H) = c$ and $u_L'(\hat{q}_L) > c$. To get the first order condition that $\hat{q}_L$ must satisfy, think about the tradeoffs in lost profit from supplying an 'inefficient' amount of electricity to L-type households and the reduction in the 'information rent' that has to be 'paid' to the H-type households to get them to select the package $\left(\hat{q}_H, \hat{T}_H\right)$.] (20 points).