Rice University

Fall Semester Final Examination 2005

ECON501 Advanced Microeconomic Theory

Writing Period: **Three Hours**

Permitted Materials: English/Foreign Language Dictionaries and non-programmable calculators

You should attempt all questions. The total points for the exam is one hundred and eighty (180).

1. **[60 Points]**

   (a) *Solve the expenditure minimization problem and derive the expenditure function for preferences represented by the utility function*

   $$u(x_1, x_2) = x_1^{\alpha} x_2^{1-\alpha}$$

   Verify the expenditure function satisfies all the requisite properties for an indirect utility function. (15 points).

   (b) *Solve the utility maximization problem and derive the indirect utility function and expenditure function for preferences represented by the utility function*

   $$U(x_1, x_2, x_3, x_4) = x_1^{1/2} x_2^{1/2} + x_3^{1/2} x_4^{1/2}.$$

   (15 points).

   Recall in lectures lectures that the equivalent variation of a change in prices and income from (p^0, w^0) to (p^1, w^1) is defined as

   $$\text{EV} = e\left(p^0, v\left(p^1, w^1\right)\right) - e\left(p^0, v\left(p^0, w^0\right)\right).$$

Further recall from lectures that if wealth is unaltered and the change in prices is caused by the imposition of commodity taxes then the deadweight loss (DWL) or excess burden of the taxes is given by

$$\text{DWL} = -\text{EV} - \sum_{l=1}^{L} t_l x_l\left(p^1, w^0\right),$$

where $t_l = p_l^1 - p_l^0$.

(c) *Briefly explain why this measure may be viewed as a deadweight loss to (social) economic efficiency* (5 points).

Suppose that an individual with preferences represented by the utility function

$$U(x_1, x_2, x_3, x_4) = x_1^{1/2} x_2^{1/2} + x_3^{1/2} x_4^{1/2}$$

initially chooses her consumption subject to a budget constraint with $p^0 = (1, 1, 1, 9)$ and $w^0 = 30$.

(d) *Calculate the DWL for this individual arising from the imposition of ad valorem taxes of 100 per cent on goods* 1 *and* 2*, that leads to the prices of good* 1 *and good* 2 *rising to* 2 *with the prices of goods* 3 *and* 4 *remaining unchanged. What would be the DWL if the rate of the ad valorem taxes on goods* 1 *and* 2 *were increased to 150 per cent raising the prices of goods* 1 *and* 2 *to* 2.5*? How about a 300 per cent ad valorem tax on goods* 1 *and* 2 *that raises their prices to* 4. (25 points).

2. **[20 Points]** Suppose that preferences are identical and homothetic. *Show that market demand for any good must be independent of the distribution of wealth. Also show that the elasticity of market demand with respect to the level of aggregate wealth must be equal to unity.*

3. **[25 Points]** Suppose there are two periods, 'today' (i.e. period 1) and 'tomorrow' (i.e. period 2), a single consumption good, and an individual called Zenon who has preferences over two-period consumption streams that are *additively separable.* In particular assume her preferences over two-period consumption streams admit a representation of the form:

$$U(x_1, x_2) = u(x_1) + \beta u(x_2), \text{ where } \beta \in (0,1) \text{ and } u(x_t) = -\frac{(x-2)^2}{2}.$$

(a) If her income today $y_1 = 1$ and she knows her income tomorrow $y_2 = 1$, and $\beta = 1/(1+r)$, where $r > 0$ is the market rate of interest at which the agent can freely borrow or lend, *solve for her optimal consumption in each period and calculate the level of discounted lifetime utility she achieves.* (10 points).

Suppose, now, that the agent again knows that income today $y_1 = 1$. However, there is uncertainty about what next period's income will be. It could be high, $y_2^H = 3/2$ or it could be low, $y_2^L = 1/2$. She believes it will be high wit probability $1/2$. Her problem now is to choose the initial period consumption x_1; her future consumption if her income tomorrow is high, x_2^H; and her future consumption if income tomorrow is low, x_2^L, in order to maximize her *discounted expected utility.*

(b) *Again, assuming that* $\beta = 1/(1+r)$*, formulate the agent's optimization problem and solve for the optimal consumption plan and her level of discounted expected utility.* (10 points).

(c) *How do you account for any difference or similarity in your answers to parts (a) and (b)?* (5 points).

4. **[25 Points]** A utility produces electricity to meet the demands of a city. The price it can charge for electricity is fixed and it must meet all demand at that price. It turns out that the amount of electricity demanded is always the same over every 24-hour period, but demand differs from day (6:00am to 6:00pm) to night (6:00pm to 6:00am). During the day, 4 units are demanded, whereas during the night only 3 units are demanded. Total output for each 24-hour period is thus always equal to 7 units. The utility produces electricity according to the production function

$$y_i = (KF_i)^{1/2}, \; i = day, night$$

where K is the size of the generating plant, and F_i is tons of fuel. The firm must build a single plant; it cannot change the plant size from day to night. Suppose a unit of plant size costs w_k per 24-hour period and a ton of fuel costs w_f. *What size plant will the utility build?*

5. **[25 Points]** A monopolist faces linear demand $p = \alpha - \beta q$ and has cost $C(q) = cq + F$, where all parameters are positive, $\alpha > c$ and $(\alpha - c)^2 > 4\beta F$.

   (a) *Solve for the monopolist's output, price and profits.* (10 points).

   (b) *Calculate the deadweight loss and show that it is positive.* (5 points).

   Now suppose the government requires the firm to set a price $\hat{p}$ that will maximize the sum of consumer and producer surplus, subject to the constraint that the firm's profit be non-negative, so that the regulation is sustainable in the long run without government payments.

   (c) *Show under this form of regulation, the firm will charge a price greater than marginal cost, and that the percentage deviation of price from marginal cost* $(\hat{p} - c)/\hat{p}$ *will be proportional to* $1/\hat{\varepsilon}$, *where* $\hat{\varepsilon}$ *is the elasticity of firm demand at the chosen price and output. Interpret your result.* (10 points).

6. **[25 Points]** An industry consists of many identical firms each with cost function $c(q) = q^2 + 1$. When there are J active firms, each firm faces an identical inverse market demand $p = 10 - 15q - (J-1)\bar{q}$ whenever an identical output of $\bar{q}$ is produced by each of the other $(J-1)$ active firms.

   (a) *With J active firms and no possibility of entry or exit, what is the short-run equilibrium output q^* each firm chooses in a symmetric equilibrium when firms act as Cournot competitors?* (10 points).

   (b) *How many firms will be active in the long run?* (15 points).