Rice University

Fall Semester Final Examination 2004

ECON501 Advanced Microeconomic Theory

Writing Period: **Three Hours**

Permitted Materials: English/Foreign Language Dictionaries and non-programmable calculators

You should answer question 1 in Part A and ***two (2)*** *out of three questions in Part B. The total points for the exam is one hundred (100). Question 1 is worth fifty (50) points and each question in Part B is worth thirty (25) points.*

PART A: Answer all Parts of Question 1.

1. [50 Points]

(a) Consider a preference relation that can be represented by the utility function

$$U(x_1, x_2, x_3, x_4) = x_1^{3/2} x_2^{3/2} + x_3^2 x_4.$$

i) Derive the uncompensated demand function for the utility maximization problem using this utility function.

ii) Verify the indirect utility function from this problem satisfies all the requisite properties for an indirect utility function.

iii) Does Roy's identity hold for this indirect utility function?
Explain or illustrate your answer.

iv) Without working out the full utility maximization problem,
show that a preference relation represented by the utility function

$$\hat{U}(x_1, x_2, x_3, x_4) = \max\left(\frac{3}{2}\ln x_1 + \frac{3}{2}\ln x_2, 2\ln x_3 + \ln x_4\right)$$

would also lead to the same uncompensated demand function.

v) Briefly explain whether or not this means that U and $\hat{U}$ represent the same preferences.

(b) Suppose, there are two periods, 'today' (i.e. period 1) and 'tomorrow' (i.e. period 2), a single consumption good, and an individual called Zeek has preferences over two-period consumption streams that are *additively separable.* In particular assume his preferences over two-period consumption streams admit a representation of the form:

$$U(x_1, x_2) = u(x_1) + u(x_2)$$

Further suppose that Zeek is also a strictly *risk-averse* expected utility maximizer. Owing to a miscalculation, Art has caused Zeek damage for which Art is legally liable. Absent the damage, Zeek's income would have been the same in both periods, say z. The damage has reduced his income in period 1 by 10%. Zeek argues that in order to make him "whole" (as well off as he would have been had Art not damaged him), Art will have to pay Zeek more in the upcoming period than the amount Art caused Zeek to lose in the previous time period, because Zeek is risk averse.

No probabilities are mentioned above, but explain why risk aversion is relevant here. Is Zeek correct in his claim that since he is risk averse, Art's period 2 payment to him must be bigger than this period 1 loss to make him whole?

(c) T-bone Pickens feeds his chickens on a mixture of soybeans and corn, depending on the prices of each. Assume the technology for 'producing' chickens is constant returns to scale. According to the data submitted by his managers, when the price of soybeans was \$10 a bushel and the price of corn was \$10 a bushel, they used 50 bushels of corn and 150 bushels of soybeans for each coop of chickens. When the price of soybeans was \$20 a bushel and the price of corn was \$10 a bushel, they used 300 bushels of corn and no soybeans per coop of chickens. When the price of corn was \$20 a bushel and the price of soybeans was \$10 a bushel, they used 250 bushels of soybeans and no corn for each coop of chickens.

i) Is there any evidence above that indicates Pickens's managers have not been minimizing costs?

ii) If Pickens's managers were always minimizing costs, briefly explain whether it is or is not possible to produce a coop of chickens using 50 bushels of soybeans and 150 bushels of corn?

PART B: Answer two (2) out of the following three questions.

2. [25 Points] Suppose there are 2 states of the world $s = 1, 2$ and a single consumption good. Assume the decision maker is a subjective expected utility maximizer with a Bernoulli utility index given by

$$u(c) = \frac{c^{1-\rho}}{1-\rho}$$

(a) What does it mean for this individual to be risk averse and what restriction do we have to place on ρ in order for this to be the case?

(b) Denoting state-contingent consumption bundles by (c_1, c_2), where c_1 represents the consumption in state 1 and c_2 represents the consumption in state 2, what implication can you draw if the bundle $(85, 45)$ is strictly preferred to $(45, 85)$? Devise a procedure that reveals the individual's subjective beliefs.

(c) Suppose your procedure reveals that the individual's subjective belief is that state 1 is three times more likely than state 2. Illustrate her indifference map in state-contingent consumption space. Show her preference relation over state-contingent consumption bundles is *homothetic*.

Now suppose the problem the individual faces is to choose a portfolio of assets subject to her budget constraint. There are only two types of assets that she can choose. The first asset whose price is 1, is a 'risk-free' asset that yields a payoff of one unit of the consumption good if either state 1 or state 2 obtains. The second asset whose price is p, only pays one unit of the consumption good if state 1 obtains. If state 2 obtains, this second asset pays out zero.

(d) Formally set up the individual's portfolio problem letting a_1 denote her demand for asset 1 and a_2 her demand for asset 2.

(e) For what range of p will $a_2(p, w) = 0$ and for what range of p will $a_2(p, w) > 0$? Will $a_1(p, w)$ ever be zero? Explain your answers.

(f) Argue that neither asset is 'inferior' for the decision maker. That is, it is never the case that at given prices if wealth goes up the demand for either of the assets goes down.

3. [**25 Points**] Consider an industry where a single output is produced using a single input. There are many technologies. Each technology can produce up to a common capacity of 1, which is *infinitesimally* small relative to the size of the market of this industry. Technologies are distinguished by a parameter b, where b is the (constant) *marginal product* of the input in this technology (up to capacity). That is, letting z, denote the quantity of input employed, the production function for technology b is:

$$F_b(z) = \begin{cases} b \times z & \text{if } z \leq 1/b \\ 1 & \text{if } z > 1/b \end{cases}$$

(a) Given the input price w and the output price p, solve the profit maximization problem of a firm with technology parameter b and derive its profit function $\pi_b(w, p)$.

Now assume technologies are distributed along $(0, \infty)$, according to the density function $h(b) = b^{-2}$. Hence the total capacity of the technologies that have a parameter b lying in the interval $[c, d]$ can be calculated by the expression

$$\int_c^d \frac{1}{b^2} db = \left[-\frac{1}{b}\right]_{b=c}^{b=d} = \frac{1}{c} - \frac{1}{d}$$

(b) Show the total industry profit is given by $\Pi(w, p) = p^2/(2w)$.

(c) Derive the industry's supply function.

(d) Derive the aggregate production function of this industry.

4. [**25 Points**] Suppose there are 1000 households who each desire one but only one video recorder and that each household is prepared to pay up to but no more than \$500. Suppose further, that there are a large number of potential manufacturers of video recorders, each with cost function

$$c(q) = q^2 + 40000$$

(a) If firms behave competitively, what is the long run equilibrium number of firms, price and quantity traded in this market?

(b) Recalling that there are a large number of identical firms who may enter this industry, determine a (sub-game perfect, pure-strategy) Nash equilibrium of the two stage game where in the first stage firms determine whether to enter the market or not (and incur the fixed cost $\$40,000$ if they do enter) and in the second stage those who have entered play a one-shot Cournot quantity-setting game. How does the aggregate welfare in this equilibrium compare with the perfectly competitive outcome computed in (a)?

(c) Design a revenue-neutral tax and rebate scheme that would implement the perfectly competitive outcome of part (a) even though producing firms act as Cournot competitors. In this context, take 'revenue-neutral' to mean that the equilibrium tax receipts equal the equilibrium rebate payments. (Hint: consider a per unit tax on production and a rebate to each household.)