FINAL 2005

Q2

$\succeq$ are identical and homothetic.

<u>Note</u> if $x \succeq y$ then $\lambda x \succeq \lambda y$

<u>So</u> if $x^* \in x(p, w)$

<u>ie</u> $p.x^* = w$ and $x^* \succeq y$ for all y s.t. $p.y \leq w$

then $\lambda x^* \succeq \lambda y$ for all y s.t. $p.(\lambda y) \leq \lambda w$

<u>ie</u> $x^* \in x(p, \lambda w)$

~~Hence $x(p, \lambda w$~~

$$x^i(p, w^i) = w^i x(p, 1)$$

So $$\sum_i x^i(p, w^i) = \sum w^i x(p,1)$$
$$= x(p,1)\left(\sum w^i\right)$$

Since $x(p, \lambda w) \equiv \lambda x(p, w)$

differentiating wrt to λ & evaluating at $\lambda=1$ $\quad \dfrac{\partial x}{\partial w}(p, w)\, w = x(p, w)$

Q3 $U(x_1, x_2) = -\frac{(x_1-2)^2}{2} - \beta\frac{(x_2-2)^2}{2}$

(a) $\max_{(x_1,x_2)} U(x_1, x_2)$ s.t. $x_1 = y_1 - s$

$x_2 = \cancel{y_2 + (1+r)s}$

$y_2 + (1+r)s$

$$\max_{s} -\frac{(y_1-2-s)^2}{2} - \frac{1}{(1+r)}\frac{(y_2+(1+r)s-2)^2}{2}$$

FOC $[y_1-2-s] - [y_2+(1+r)s-2] = 0$

So $[y_1 - y_2] = 2(1+r)s$

So $s = \frac{[y_1-y_2]}{2[1+r]}$

$s = 0$ so $x_1 = x_2 = 1$, $\cancel{U(1,1)}$

$U(1,1) = -\frac{1}{2} - \frac{(1+r)}{2} = -\frac{(2+r)}{2}$.

(b) $$\max_{\langle x_1, x_2^H, x_2^L \rangle} -\frac{(x_1-2)^2}{2} - \beta\left[\frac{(x_2^H-2)^2}{4} + \frac{(x_2^L-2)^2}{4}\right]$$

s.t. $$x_1 = y_1 \cancel{*} - s$$

$$x_2^H = y_1^H + (1+r)s$$

$$x_2^L = y_1^L + (1+r)s$$

~~FOC~~ $$\max_{\langle s \rangle} -\frac{[y_1-2-s]^2}{2} - \frac{1}{(1+r)}\left[\frac{(y_2^H+(1+r)s-2)^2}{4} + \frac{(y_2^L+(1+r)s-2)^2}{4}\right]$$

FOC $$[y_1-2-s] - \frac{1}{2}[y_2^H - 2 + (1+r)s] - \frac{1}{2}[y_2^L - 2 + (1+r)s] = 0$$

So ~~s =~~ $$y_1 - \frac{1}{2}y_2^H - \frac{1}{2}y_2^L = 2(1+r)s$$

Hence $$s^* = \frac{[y_1 - E[y_2]]}{2[1+r]}$$

Q4

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$$\min_{\langle K, F_D, F_N \rangle} \quad \omega_k K + \omega_f (F_N + F_D)$$

$$\text{s.t.} \quad (K F_D)^{1/2} = 4$$

$$\text{and} \quad (K F_N)^{1/2} = 3$$

So $F_D = 16/K$

$F_N = 9/K$

$$\min_{\langle K \rangle} \quad \omega_k K + W_f \left(\frac{25}{K} \right)$$

FOC $\quad W_k - W_f \dfrac{25}{K^2} = 0$

$$\rightarrow \quad K^2 = 5 \sqrt{\frac{W_f}{W_k}}$$