Rice University
2006 Fall Semester Final Exam

Introduction to Game Theory (Econ 340)

December 2006

- *Permitted Materials: Nonprogrammable Calculators, English and Foreign Language Dictionaries*
- *Duration: 3 hours.*
- There are **120** total points available. The exam has **4** questions and **6** pages. **Please put each question into a separate blue book.**

Question 1. **[35 total points. Use blue book 1.]** State whether each of the following *italized* claims is true or false (or can not be determined). For each, explain your answer in (at most) **one** short paragraph**.** Each part is worth **7** points**, of which 6 points are for the explanation.** Explaining an example or a counter-example is sufficient. Absent this, a nice concise intuition is sufficient: you do not need to provide a formal proof. Points will be deducted for incorrect explanations.

(a) Recall the two-player 'grade game' in which each player simultaneously chooses either α or β. The *outcomes* from their action choices are summarized in the table below:

		Colin	
		α	β
Rowena	α	(B^-, B^-)	(A, C)
	β	(C, A)	(B^+, B^+)

For example, if Rowena plays β and Colin plays α then Rowena gets the grade C and Colin gets the grade A.

Suppose Rowena is rational.

Then if Rowena is an 'evil git' or if Rowena is an 'indignant angel' and Rowena knows Colin is an 'evil git' then Rowena will play β.

(b) *If a game has a strict Nash equilibrium in pure strategies then it cannot have a mixed strategy equilibrium.*

(c) Consider the following payoff matrix for a two-person symmetric game.

		Column	
		L	R
Row	L	10^6, 10^6	0, 0
	R	0, 0	1, 1

Both (L, L) *and* (R, R) *are symmetric Nash equilibria of the game but only the strategy* L *is evolutionary stable.*

(d) *In the alternating-offer bargaining game, if there are just two stages, the equilibrium payoff of the person who gets to make the first offer is decreasing in the degree of patience of the other player (that is, if you increase the discount factor,* δ *of player* 2*, then you decrease player* 1*'s equilibrium payoff).*

(e) *In a subgame perfect equilibrium of a finitely repeated game, at each stage along the equilibrium the action profile chosen by the players must constitute a Nash Equilibrium of the one-shot stage game.*

USE BLUE BOOK 1

Question 2. [**30 total points**] **"Hold Up"**

The CEOs of two firms — let's call them Abby and Bobby — are considering devoting resources to a joint project. For the project to succeed, it requires a *total* contribution of \$9 billion. Abby's company can make contributions toward the project in period 1 and period 3. Bobby's company can only contribute in period 2. Let a_1 be the contribution Abby chooses to make in period 1. Let a_3 be the contribution Abby chooses to make in period 3. And let b_2 be the contribution that Bobby makes in period 2.

If the project is successful, whenever it is successful, then Abby's company makes \$10 billion profit. If the project is successful and that success occurs in period 1 or period 2, then it is also worth \$10 billion to Bobby's company. But if the project is only successful in period 3, then it is only worth \$5 billion to Bobby's company. Assume that all contributions in each period are observable, and that contributions cannot be negative. Here is a payoff table (in \$billions)

Abby's payoff	Bobby's payoff	
$-a_1 - a_3$	$-b_2$	if $a_1 + b_2 + a_3 < 9$
$10 - a_1 - a_3$	$10 - b_2$	if $a_1 + b_2 \geq 9$
$10 - a_1 - a_3$	$5 - b_2$	if $a_1 + b_2 < 9 \leq a_1 + b_2 + a_3$

(a) [**10 points**] Suppose that Abby has contributed \$2 billion in period 1 (i.e., $a_1 = 2$). How would you expect the game to proceed? Explain.

(b) [**10 points**] How would you expect the game to proceed from the start? Explain.

Suppose now that, in period 0, prior to anyone making any contributions to the joint project, Bobby can invest in another project that would increase the profit of the joint project provided the joint project succeeds in periods 1 or 2 (that is, before period 3). Bobby can invest \$0, \$1 million, or \$2 million in the other project, yielding payoffs to him of $\$(10 - b_2)$billion; $\$(12 - b_2)$billion - \$1 million; and $\$(14 - b_2)$billion - \$2 million respectively if $a_1 + b_2 \geq 9$. The other project has no effect on the value of the joint project for Bobby if the project only succeeds in period 3, and it has no effect on the value of the project for Abby in any case.

Assume that the amount that Bobby invests in period 0 is observable to all before contributions are made toward the joint project, and that everyone understands the effects on payoffs.

(c) [**10 points**] How much should Bobby invest in period 0? Explain.

USE BLUE BOOK 3

Question 3. [**35 total points**] **"Trade Wars"**

New Holland exports widgets to Old Sheep Land. There is a perfectly competitive market for both consumers and producers of widgets in the two countries. In trade theory, the 'optimal tariff' argument states that a government of a large economy may be able to improve domestic welfare by exploiting the country's market power in international trade by imposing a trade tax (that is, a tariff if the country imports the good and an export tax if it exports the good) which improves the country's terms of trade. Of course if both countries impose trade taxes, the volume of trade relative to the free trade outcome falls and if the absolute sizes of the price elasticities of both the export demand and import supply are similar the result is worse for *both* countries than the free-trade outcome.

To simplify the analysis suppose the optimal tariff for Old Sheep Land is independent of the level of export tax set by New Holland and similarly, the optimal export tax for New Holland is independent of the level of tariff set by Old Sheep Land. For both countries let 0 and T denote the zero trade tax and optimal trade tax respectively. Suppose the payoff matrix is as follows:

		Old Sheep 0	Old Sheep T
New Holland	0	3, 3	1, 4
	T	4, 1	2, 2

(a) [**4 points**] What is the equilibrium of this trade-tax setting game if played once? Make sure to show why it is an equilibrium.

(b) [**7 points**] Given that the one period discount factor of each country's government is $\delta \in (0, 1)$, what can be sustained as a subgame perfect equilibrium if the trade-tax setting stage game is played five times? Make sure you explain (at least briefly) the reasoning behind your answer.

(c) [**12 points**] How large must δ be in order that free-trade can be sustained as a subgame perfect equilibrium of the infinitely repeated game? In particular write down a subgame perfect equilibrium strategy profile that sustains free-trade for a sufficiently large δ and show that it is indeed subgame perfect.

Question Continued on next page.

Now suppose that each government's horizon is only two periods and the (common) discount factor is 1. Moreover suppose that in either of these two periods a government has the option of setting its trade tax so high that the two countries revert to *autarky* for that period (that is, there is *no* trade between the two countries). Suppose the payoff to each country in autarky is zero.

(d) [**12 points**] Modify the payoff matrix of the stage game to include the option of a country setting a trade tax that induces autarky (label this action A). Analyse whether it is possible to sustain free-trade in either period as part of a subgame perfect equilibrium of the once repeated (that is, two period) game.

USE BLUE BOOK 3

USE BLUE BOOK 4

Question 4. [**20 total points.**] **"Research Proposals as Signals".**

A researcher is preparing a proposal document for a research project that she wishes a government funding agency to finance. The productivity of the researcher, denoted by p, is either 2, (that is, high) or 1 (that is, low). The researcher who knows what her productivity is, chooses the level of "quality", $q \geq 0$, for the proposal document and the amount of funding, $g \geq 0$, that she says is needed for the project to go ahead. The agency after observing the quality of the proposal document, decides whether to fund the project or not. Assume that the agency either provides the full amount of funding requested or rejects the project outright. If the proposal is successful (that is, the project is funded by the agency) then the payoff to the researcher is $g - (q/p)$ and the payoff to the research agency is $p - g$. If the proposal is unsuccessful then the payoff to the researcher is $-(q/p)$ and the payoff to the agency is zero.

(a) [**4 points.**] Suppose that there is complete information so that the agency does observe the researcher's productivity at the same time as observing the quality of the research proposal. What level of quality will be chosen by each type of researcher? Briefly explain your answer.

(b) [**16 points.**] Suppose now that the agency does not observe the researcher's productivity so that there is asymmetric information. In particular, suppose that the agency has no specific information at all about the researcher except that it is equally likely her productivity may be high or may be low, and that the researcher is aware of the agency's lack of knowledge about her. How can this situation be analysed using game theoretic techniques and concepts? Provide an example of a 'pooling' equilibrium and an example of a 'separating' equilibrium for this game. Be careful to show that each of your examples satisfies all the conditions required for it to be an equilibrium.

USE BLUE BOOK 4