

Rice University
2006 Fall Semester Final Exam
Introduction to Game Theory (Econ 340)

December 2006

- *Permitted Materials: Nonprogrammable Calculators, English and Foreign Language Dictionaries*
- *Duration: 3 hours.*
- There are **120** total points available. The exam has **4** questions and **6** pages.
Please put each question into a separate blue book.

Question 1. [35 total points. Use blue book 1.] State whether each of the following *italized* claims is true or false (or can not be determined). For each, explain your answer in (at most) **one** short paragraph. Each part is worth **7** points, **of which 6 points are for the explanation**. Explaining an example or a counter-example is sufficient. Absent this, a nice concise intuition is sufficient: you do not need to provide a formal proof. Points will be deducted for incorrect explanations.

- (a) Recall the two-player ‘grade game’ in which each player simultaneously chooses either α or β . The *outcomes* from their action choices are summarized in the table below:

| | | | |
|--------|----------|--------------|--------------|
| | | Colin | |
| | | α | β |
| Rowena | α | (B^-, B^-) | (A, C) |
| | β | (C, A) | (B^+, B^+) |

For example, if Rowena plays β and Colin plays α then Rowena gets the grade C and Colin gets the grade A .

Suppose Rowena is rational.

Then if Rowena is an ‘evil git’ or if Rowena is an ‘indignant angel’ and Rowena knows Colin is an ‘evil git’ then Rowena will play β .

ANS: False. If both palyers are evil gits, the payoff matrix for this game would take something like the following form:

| | | | |
|--------|----------|----------|---------|
| | | Colin | |
| | | α | β |
| Rowena | α | 1 | 0 |
| | β | 3 | 2 |

For each player, strategy β is strictly dominated by strategy choice α , and will never be played. The result in this case will be (α, α) being played.

If Rowena is an indignant angel and she knows Colin is an evil git, she can still out cross the possibility of Colin ever playing β (since she knows a rational player will never play a strictly dominated strategy). Thus she is left between the play being (α, α) or (β, α) . Because her payoff associated with (α, α) is higher than that associated with (β, α) , she will not play β either.

- (b) *If a game has a strict Nash equilibrium in pure strategies then it cannot have a mixed strategy equilibrium.*

False. Consider the “Battle-of-the-Sexes” game:

| | | | |
|-------|---|--------|--------|
| | | Pl. 2 | |
| | | ℓ | r |
| Pl. 1 | U | 1 2 | 0 0 |
| | D | 0 0 | 2 1 |

In this game, both (U, ℓ) and (D, r) are strict Nash equilibria in pure strategies (because for both $i = 1, 2$ $u_i(\hat{s}, \hat{s}) > u_i(s', \hat{s})$, holds). Nevertheless, this game has a mixed strategy equilibrium, namely $[(2/3, 1/3), (1/3, 2/3)]$.

(c) Consider the following payoff matrix for a two-person symmetric game.

| | | | |
|-----|---|------------------|--------|
| | | Column | |
| | | L | R |
| Row | L | 10^6 10^6 | 0 0 |
| | R | 0 0 | 1 1 |

Both (L, L) and (R, R) are symmetric Nash equilibria of the game but only the strategy L is evolutionary stable.

False. Recall the definition for \hat{s} to be evolutionary stable is that there exists $\bar{\varepsilon} > 0$, such that for all $\varepsilon < \bar{\varepsilon}$,

$$(1 - \varepsilon) \underbrace{[u(\hat{s}, \hat{s}) - u(s', \hat{s})]}_A + \varepsilon \underbrace{[u(\hat{s}, s') - u(s', s')]}_B > 0$$

If $A > 0$, then it follows that such an $\bar{\varepsilon}$ exists (since for sufficiently small ε , the expression $(1 - \varepsilon)A$ will ensure the inequality holds no matter what the sign of B is). Let $\hat{s} = R$ and $s' = L$, then $A = u(R, R) - u(L, R) = 1 - 0 > 0$, thus R is evolutionary stable as well.

(d) In the alternating-offer bargaining game, if there are just two stages, the equilibrium payoff of the person who gets to make the first offer is decreasing in the degree of patience of the other player (that is, if you increase the discount factor, δ of player 2, then you decrease player 1’s equilibrium payoff).

True. In the second stage, both players know the proposer for that stage (i.e. player 2) will get everything (if we assume a responder accepts a proposal when indifferent between accepting or rejecting it). So in the first stage, the proposer for that stage (i.e. player 1) knows what he offers to the responder (i.e. player 2) must be worth

to her at least as much as getting everything in the next period. That is, if she anticipates getting 1 in the next period, she must be offered δ_2 (where δ_2 is her discount factor) in the first period. Hence this backward induction reasoning leads player to 1 to offer the split, $1 - \delta_2$ for himself and δ_2 for 2. As the discount factor for player 2 increases, clearly 1's equilibrium payoff decreases.

- (e) *In a subgame perfect equilibrium of a finitely repeated game, at each stage along the equilibrium the action profile chosen by the players must constitute a Nash Equilibrium of the one-shot stage game.*

False. Consider the following game:

| | | | | |
|-------|-----|--------|-----|-----|
| | | Pl. 2 | | |
| | | ℓ | c | r |
| Pl. 1 | U | 3 | 4 | 0 |
| | M | 1 | 2 | 0 |
| | D | 0 | 0 | 0 |

The only NE in the one-shot game are (M, c) and (D, r) . However, using the strategy described below, we can show that in the first stage of the 2 period version of the game (i.e. once repeated) (U, ℓ) can be sustained as part of an SPE.

Consider the strategy profile:

Player 1. In period 1, play U . In period 2, play M if (and only if) (U, ℓ) was played in period 1, otherwise play D .

Player 2. In period 1, play ℓ . In period 2, play c if (and only if) (U, ℓ) was played in period 1, otherwise play r .

The idea is that if there are multiple equilibria in the stage game then it may be possible to sustain non-stage game equilibrium play in the first stage of the once-repeated game by using the equilibria of the stage-game as rewards and/or punishments in the second stage.

Question 2. [30 total points] “Hold Up”

The CEOs of two firms — let's call them Abby and Bobby — are considering devoting resources to a joint project. For the project to succeed, it requires a *total* contribution of \$9 billion. Abby's company can make contributions toward the project in period 1 and period 3. Bobby's company can only contribute in period 2. Let a_1 be the contribution Abby chooses to make in period 1. Let a_3 be the contribution

Abby chooses to make in period 3. And let b_2 be the contribution that Bobby makes in period 2.

If the project is successful, whenever it is successful, then Abby's company makes \$10 billion profit. If the project is successful and that success occurs in period 1 or period 2, then it is also worth \$10 billion to Bobby's company. But if the project is only successful in period 3, then it is only worth \$5 billion to Bobby's company. Assume that all contributions in each period are observable, and that contributions cannot be negative. Here is a payoff table (in \$billions)

| Abby's payoff | Bobby's payoff | |
|------------------|----------------|---|
| $-a_1 - a_3$ | $-b_2$ | if $a_1 + b_2 + a_3 < 9$ |
| $10 - a_1 - a_3$ | $10 - b_2$ | if $a_1 + b_2 \geq 9$ |
| $10 - a_1 - a_3$ | $5 - b_2$ | if $a_1 + b_2 < 9 \leq a_1 + b_2 + a_3$ |

- (a) [10 points] Suppose that Abby has contributed \$2 billion in period 1 (i.e., $a_1 = 2$). How would you expect the game to proceed? Explain.

ANS: $a_1 = 2$. Note that Abby should always, if the game reaches period 3, invest the remaining amount, because \$10 billion profit will be better than not investing (i.e. net payoff = $0 - a_1$) even if she fronts all \$9 billion in cost (i.e. net payoff = \$1 billion.)

Bobby should thus expect in period 3, Abby will contribute $a_3 = 9 - a_1 - b_2 = 7 - b_2$. So if Bobby invest all \$7 billion today, he gets next payoff $\$10 - 7 = \3 billion. But if he invests $b_2 < 7$ he anticipates a payoff of $\$5 - b_2$ which is maximized by setting $b_2 = 0$.

So I predict given Abby has played $a_1 = 2$ in period 1, Bobby will choose $b_2 = 0$ in period 2 and Abby will then play $a_3 = 7$ in period 3.

- (b) [10 points] How would you expect the game to proceed from the start? Explain.

ANS. From the analysis in part (a) we see that Abby can induce Bobby to contribute in period 2 only if $10 - b_2 \geq 5 - 0$. That is, she can only induce Bobby to contribute $b_2 \leq 5$. To get Bobby to contribute $b_2 = 5$, she must invest in period 1, $a_1 = 4$ (so we will have $a_1 + b_2 = 9$). This is the smallest amount that Abby can invest in period 1, and still induce Bobby to contribute $b_2 = 5$, because he is indifferent between investing $b_2 = 5$ and getting $\$10 - 5 = \5 billion or investing nothing and (correctly) anticipating that Abby will invest the remaining amount needed to complete the project in period 3 netting Bobby \$5 billion.

So I predict given Abby will play $a_1 = 4$ in period 1, Bobby will choose $b_2 = 5$ in period 2 and Abby will then play $a_3 = 0$ in period 3.

Suppose now that, in period 0, prior to anyone making any contributions to the joint project, Bobby can invest in another project that would increase the profit of the joint project provided the joint project succeeds in periods 1 or 2 (that is, before period 3). Bobby can invest \$0, \$1 million, or \$2 million in the other project, yielding payoffs to him of $\$(10 - b_2)$ billion; $\$(12 - b_2)$ billion - \$1 million; and $\$(14 - b_2)$ billion - \$2 million respectively if $a_1 + b_2 \geq 9$. The other project has no effect on the value of the joint project for Bobby if the project only succeeds in period 3, and it has no effect on the value of the project for Abby in any case.

Assume that the amount that Bobby invests in period 0 is observable to all before contributions are made toward the joint project, and that everyone understands the effects on payoffs.

(c) [10 points] How much should Bobby invest in period 0? Explain.

ANS: If Bobby invests nothing in period 0 then the play will be as we described in part (b) and his payoff will be \$5 billion on the joint project.

If he invests \$1 million, his payoff changes to $\$12 - b_2 - 0.001$ if the joint project is successful in period 2 and $\$5 - b_2 - 0.001$ if the project does not succeed until period 3. So Bobby is willing to invest up to \$7 billion in period 2 to make the project succeed in period 2, since

$$12 - b_2 - 0.001 \geq 5 - 0.001 \Rightarrow b_2 \leq 7$$

Knowing this, Abby need only invest \$2 billion in period 1, because she wants Bobby to pay for as much of the joint project. Hence in this scenario, $a_1 = 2$, $b_2 = 7$, and $a_3 = 0$, yielding a payoff to Bobby of $5 - 0.001 < 5$.

If he invest \$2 million, his payoff changes to $\$14 - b_2 - 0.002$ if the joint project is successful in period 2 and $\$5 - b_2 - 0.002$ if the project does not succeed until period 3. So Bobby is willing to invest up to \$9 billion in period 2 to make the project succeed in period 2, since

$$14 - b_2 - 0.001 \geq 5 - 0.001 \Rightarrow b_2 \leq 9$$

Knowing this, Abby can now completely free ride in period 1, because she knows Bobby is willing to pay for all of the joint project. Hence in this scenario, $a_1 = 0$, $b_2 = 9$, and $a_3 = 0$, yielding a payoff to Bobby of $5 - 0.002 < 5$.

Hence, Bobby should not invest any money in period 0. The reason is that Abby gets to see Bobby's period 0 decision and she knows what it implies for his profits for the joint project. Hence his investment in period 0, just allows her to reduce her contribution towards the joint

project by the amount that his investment increases the payoff to him of achieving an early success to the joint project. Which means he reaps none of the benefit of increasing that payoff, but has to incur the (relatively modest) cost of that investment.

Question 3. [35 total points] “Trade Wars”

New Holland exports widgets to Old Sheep Land. There is a perfectly competitive market for both consumers and producers of widgets in the two countries. In trade theory, the ‘optimal tariff’ argument states that a government of a large economy may be able to improve domestic welfare by exploiting the country’s market power in international trade by imposing a trade tax (that is, a tariff if the country imports the good and an export tax if it exports the good) which improves the country’s terms of trade. Of course if both countries impose trade taxes, the volume of trade relative to the free trade outcome falls and if the absolute sizes of the price elasticities of both the export demand and import supply are similar the result is worse for *both* countries than the free-trade outcome.

To simplify the analysis suppose the optimal tariff for Old Sheep Land is independent of the level of export tax set by New Holland and similarly, the optimal export tax for New Holland is independent of the level of tariff set by Old Sheep Land. For both countries let 0 and T denote the zero trade tax and optimal trade tax respectively. Suppose the payoff matrix is as follows:

| | | | |
|-------------|-----|-----------|-----|
| | | Old Sheep | |
| | | 0 | T |
| New Holland | 0 | 3 | 4 |
| | T | 1 | 2 |

- (a) [4 points] What is the equilibrium of this trade-tax setting game if played once? Make sure to show why it is an equilibrium.

ANS: This is like the prisoners’ dilemma. For both players, T strictly dominates 0 (since $4 > 3$ and $2 > 1$) so the only Nash equilibrium is (T, T) , that is, in which each is playing its strictly dominant strategy.

- (b) [7 points] Given that the one period discount factor of each country’s government is $\delta \in (0, 1)$, what can be sustained as a subgame perfect equilibrium if the trade-tax setting stage game is played five times? Make sure you explain (at least briefly) the reasoning behind your answer.

As this is a finite repeated game we can use backward induction, that is, start from the end of the game and work back up to the beginning.

So consider how the game will be played in a SPE in the last period. Let u_N (respectively, u_O) be the payoff of New Holland (respectively, Old Sheep Land) from the first four plays of the game. The payoffs from the remaining stage are thus given by:

| | | | |
|-------------|-----|------------------------|------------------------|
| | | Old Sheep | |
| | | 0 | T |
| New Holland | 0 | $u_O + 3$ $u_N + 3$ | $u_O + 4$ $u_N + 1$ |
| | T | $u_O + 1$ $u_N + 4$ | $u_O + 2$ $u_N + 2$ |

Hence no matter what the history of play has been in the first four periods, for both players, T strictly dominates 0 (since $u_i + 4 > u_i + 3$ and $u_i + 2 > u_i + 1$) so the only equilibrium play in the final stage is (T, T) , that is, in which each is playing its strictly dominant strategy.

But now when we consider play in the fourth stage, both players know that in the fifth stage both will play (T, T) no matter what the history of play has been up to this point and no matter what they do in the fourth stage. Hence if we let u_N (respectively, u_O) be the payoff of New Holland (respectively, Old Sheep Land) from the first three plays of the game. The SPE payoff resulting from their action choice in period 4 is given by:

| | | | |
|-------------|-----|--|--|
| | | Old Sheep | |
| | | 0 | T |
| New Holland | 0 | $u_O + 3 + \delta 2$ $u_N + 3 + \delta 2$ | $u_O + 4 + \delta 2$ $u_N + 1 + \delta 2$ |
| | T | $u_O + 1 + \delta 2$ $u_N + 4 + \delta 2$ | $u_O + 2 + \delta 2$ $u_N + 2 + \delta 2$ |

Hence no matter what the history of play has been in the first three periods, for both players, T will yield a higher SPE payoff than 0 whatever the other country does (since $u_i + 4 + \delta 2 > u_i + 3 + \delta 2$ and $u_i + 2 + \delta 2 > u_i + 1 + \delta 2$) so the only play in the fourth stage that can be sustained in a subgame perfect equilibrium is (T, T) .

But similar reasoning means the only play in the third stage or the second stage or the first stage that can be sustained in a SPE is (T, T) . Hence the only SPE is one in which each player plays T at every stage irrespective of the history of play up to that stage.

- (c) [12 points] How large must δ be in order that free-trade can be sustained as a subgame perfect equilibrium of the infinitely repeated game? In particular write

down a subgame perfect equilibrium strategy profile that sustains free-trade for a sufficiently large δ and show that it is indeed subgame perfect.

“Grim-trigger” strategy: Both players follow the strategy –

- play 0 if $(0, 0)$ has been played in every period up to this point.
- play T otherwise.

In order to check that this is SPE it is sufficient (by the one-step deviation property) to check that there are no profitable one-step deviations either on or off the equilibrium path.

Cooperation Phase: $(0, 0)$ forever.

If play equil get: $3 + \delta 3 + \delta^2 3 + \delta^3 3 + \dots = 3 / (1 - \delta)$

One-step deviation: 4 (from $(T, 0)$) $+ \delta 2 + \delta^2 2 + \dots = 2 + 2 + \delta 2 + \delta^2 2 + \dots = 2 + 2 / (1 - \delta)$

Hence require

$$\frac{3}{(1 - \delta)} \geq \frac{2 - 2\delta + 2}{(1 - \delta)} \Rightarrow \delta \geq \frac{1}{2}.$$

Punishment Phase: (T, T) forever after.

If play according to strategy get: $2 + \delta 2 + \delta^2 2 + \dots = 2 / (1 - \delta)$

One-step deviation: $1 + \delta 2 + \delta^2 2 + \dots = -1 + 2 / (1 - \delta)$

So payoff from following equilibrium strategy in the punishment phase will always exceed payoff from 1-step deviation for all values of δ in $(0, 1)$.

Thus the “grim-trigger” strategy profile can sustain free-trade in every period as the SPE outcome if $\delta \geq 1/2$.

Now suppose that each government’s horizon is only two periods and the (common) discount factor is 1. Moreover suppose that in either of these two periods a government has the option of setting its trade tax so high that the two countries revert to *autarky* for that period (that is, there is *no* trade between the two countries). Suppose the payoff to each country in autarky is zero.

- (d) [12 points] Modify the payoff matrix of the stage game to include the option of a country setting a trade tax that induces autarky (label this action A). Analyse whether it is possible to sustain free-trade in either period as part of a subgame perfect equilibrium of the once repeated (that is, two period) game.

Modified game:

| | | | | |
|-------------|-----|-----------|-----|-----|
| | | Old Sheep | | |
| | | 0 | T | A |
| New Holland | 0 | 3 | 4 | 0 |
| | T | 3 | 1 | 0 |
| | A | 1 | 2 | 0 |
| | | 0 | 0 | 0 |
| | | 0 | 0 | 0 |

The Nash equilibria in this new game are: (T, T) and (A, A) .

Consider the following strategy:

In period 1, play action 0

In period 2, play T if $(0, 0)$ was played in period 1, otherwise play A .

If both countries play this strategy, then free trade will be sustained in the first period.

To check that this is a SPE:

In the second stage, the strategy profile requires that either (T, T) or (A, A) be played. Both are NE of the stage game, for each possible second-stage game, since the play prescribed is Nash, neither player has a strict incentive to deviate.

In the first stage: need to check that the payoff from sticking to the putative equilibrium is no less than that of deviating.

Payoff on equilibrium path: 3 in period 1 (from $(0, 0)$) + 2 in period 2 (from (T, T)) = 5

Greatest payoff from deviating: 4 in period 1 (from $(T, 0)$) + 0 in period 2 (from (A, A)) = 4

Thus players have no strict incentive to deviate. We have thus shown that the strategy profile described above induces a NE in every sub-game and is thus an SPE.

USE BLUE BOOK 4

Question 4. [20 total points.] “Research Proposals as Signals”.

A researcher is preparing a proposal document for a research project that she wishes a government funding agency to finance. The productivity of the researcher, denoted by p , is either 2, (that is, high) or 1 (that is, low). The researcher who knows what her productivity is, chooses the level of “quality”, $q \geq 0$, for the proposal document and the amount of funding, $g \geq 0$, that she says is needed for the project to go ahead. The agency after observing the quality of the proposal document, decides whether to fund the project or not. Assume that the agency either provides the

full amount of funding requested or rejects the project outright. If the proposal is successful (that is, the project is funded by the agency) then the payoff to the researcher is $g - (q/p)$ and the payoff to the research agency is $p - g$. If the proposal is unsuccessful then the payoff to the researcher is $-(q/p)$ and the payoff to the agency is zero.

- (a) [4 points.] Suppose that there is complete information so that the agency does observe the researcher's productivity at the same time as observing the quality of the research proposal. What level of quality will be chosen by each type of researcher? Briefly explain your answer.

ANS. If $p = 1$. Agency is 2nd mover, and will only fund if $p - g \geq 0 \Rightarrow 1 - g \geq 0 \Rightarrow g \leq 1$. Knowing this, the researcher will ask for $g = 1$ to maximize payoff. Because agency can observe productivity of researcher, there is no reason to write a quality proposal, so $q = 0$ provides no cost. So predict, $g = 1, q = 0$. Agency funds project and payoffs are $U_R = 1 - 0/1 = 1, U_A = 1 - 1 = 0$.

If $p = 2$. Agency is 2nd mover, and will only fund if $p - g \geq 0 \Rightarrow 2 - g \geq 0 \Rightarrow g \leq 2$. Knowing this, the researcher will ask for $g = 2$ to maximize payoff. Because agency can observe productivity of researcher, there is no reason to write a quality proposal, so $q = 0$ provides no cost. So predict, $g = 2, q = 0$. Agency funds project and payoffs are $U_R = 2 - 0/2 = 2, U_A = 2 - 2 = 0$.

- (b) [16 points.] Suppose now that the agency does not observe the researcher's productivity so that there is asymmetric information. In particular, suppose that the agency has no specific information at all about the researcher except that it is equally likely her productivity may be high or may be low, and that the researcher is aware of the agency's lack of knowledge about her. How can this situation be analysed using game theoretic techniques and concepts? Provide an example of a 'pooling' equilibrium and an example of a 'separating' equilibrium for this game. Be careful to show that each of your examples satisfies all the conditions required for it to be an equilibrium.

Pooling Equilibrium

Agency believes it can draw no inference about the researcher's productivity from the proposal. So given its beliefs the expected productivity of any researcher who submits a proposal is taken to be the population average, that is $0.5 \times 1 + 0.5 \times 2 = 3/2$. Given this belief, best response for agency to accept any proposal with $g \leq 3/2$.

ANS. If $p = 1$. Researcher sets $q = 0$, since in equilibrium his productivity will be treated as if it is average (i.e. $3/2$) irrespective of how high a quality it is. Since $q > 0$, involves a cost but returns no

benefit, $q = 0$ is optimal. And since his productivity is treated as if it the population average, any proposal with $g \leq 3/2$ will be accepted and any with $g > 3/2$ will be rejected. So researcher maximizes his payoff by requesting $g = 3/2$.

ANS. If $p = 2$. Researcher sets $q = 0$, since in equilibrium his productivity will be treated as if it is average (i.e. $3/2$) irrespective of how high a quality it is. Since $q > 0$, involves a cost but returns no benefit, $q = 0$ is optimal. And since his productivity is treated as if it the population average, any proposal with $g \leq 3/2$ will be accepted and any with $g > 3/2$ will be rejected. So researcher maximizes his payoff by requesting $g = 3/2$.

As both types of researchers choose $q = 0$ and $g = 3/2$, it is indeed consistent with the researchers' behavior for the agency to assess the expected productivity of any researcher who submits a proposal to be $3/2$.

Separating Equilibrium

The quality of the proposal is now a signal to the agency as to whether or not the researcher has productivity 1 or 2. Consider following strategy profile:

- (a) Agency employs following strategy. If it receives proposal with $q \geq 1$ assumes researcher has productivity 2 and so will fund any proposal with $g \leq 2$. If it receives proposal with $q < 1$, assumes researcher has productivity 1 and so will fund any proposal with $g \leq 1$.
- (b) Researcher with $p = 1$. Submits proposal with $q = 0$ and $g = 1$.
- (c) Researcher with $p = 2$. Submits proposal with $q = 1$ and $g = 2$.

Need to check that everyone is playing a best response given what everyone else is doing AND that the agency's inference is consistent with the strategy choice of the researchers.

Agency

Note since researcher with $p = 1$, submits proposal with $q = 0$ and researcher with $p = 2$ submits proposal with $q = 1$, it is consistent for agency to assume any proposal with $q \geq 1$ comes from productivity $p = 2$ researcher and any proposal with $q < 1$ comes from productivity $p = 1$ researcher. And so best response for researcher to fund proposal $q = 0$ and $g = 1$, (payoff is $1 - 1 \geq 0$) and to fund proposal $q = 1$ & $g = 2$ (payoff is $2 - 2 \geq 0$).

Researcher with $p = 1$

If he follows the above strategy his payoff is $1 - 0 = 1$. Only way it can get more funding is to submit proposal with $q \geq 1$. Best deviation is then to submit a proposal like those submitted by a researcher with $p = 2$ in the equilibrium. But this yields only a payoff of $2 - 1/1 = 1$. Hence no strict incentive for researcher with $p = 1$, to deviate. Even if he is mistaken for a researcher with $p = 2$, the cost of submitting such a high quality proposal offsets completely the greater funding he can obtain from the funding agency.

Researcher with $p = 2$

If he follows the above strategy his payoff is $2 - 1/2 = 3/2$. He cannot get more funding, and asking for less with the same level of quality of proposal just reduces his payoff. But he knows he can get funding of 1 without any effort by submitting a proposal like those submitted by a researcher with $p = 1$, that is a proposal with $q \geq 0$ and $g = 1$. But this best deviation with a different quality of proposal (in fact zero quality) yields only a payoff of $1 - 0/2 = 1 < 3/2$. Hence there is no strict incentive for the researcher with $p = 1$, to deviate. Even if he is mistaken for a researcher with $p = 2$, the cost of submitting such a high quality proposal offsets completely the greater funding he can obtain from the funding agency.