If e_r is constant its growth rate is zero, so that this may be rearranged as

$$\frac{\dot{e_n}}{e_n} = \frac{\dot{P_f}}{P_f} - \frac{\dot{P_d}}{P_d}.$$

This is positive if the foreign inflation rate exceeds the domestic, indicating that the domestic currency appreciates.

6.35 (a) Taking logs and derivatives, the relationship between the growth rates is

$$\frac{\dot{y}}{y} = \frac{\dot{Y}}{Y} - \frac{\dot{P}}{P}.$$

(b) i. For $n - 1 = 61$, the average growth rates are as follows.

Real GNP	$(\log 720.0 - \log 116.8)/61 = 0.0298$
Nominal GNP	$(\log 974.126 - \log 33.4)/61 = 0.0553$
GNP Deflator	$(\log 135.3 - \log 29.1)/61 = 0.0252$

They are approximately related by the growth identity of part (a):

$$0.0298 = 0.0553 - 0.0252.$$

ii. For $n - 1 = 70$, the average growth rates are as follows.

Real wage	$(\log 7008 - \log 1948)/70 = 0.0183$
Nominal wage	$(\log 8150 - \log 487)/70 = 0.0403$
CPI	$(\log 116.3 - \log 25.0)/70 = 0.0220$

They are related by the growth identity of part (a):

$$0.0183 = 0.0403 - 0.0220.$$

6.36 (a) The 1994-97 log-differences are as follows.

	SST Index	Decomposition: Rate	Gap	$1+G$
Canada	0.172	0.058	0.114	−0.011
United States	−0.172	−0.098	−0.078	0.007

The equality is satisfied, up to an element of rounding error arising from the fact that the 1997 values in Table 6.10 are given to only three decimal places accuracy.

(b) Yes, Osberg's conclusions are consistent with the data.

Examination Questions

1. Consider a log-log regression model. Indicate by circling T or F whether the following statements are true or false.

T **F** The least squares slope estimator is not affected by changes in units of measure.

T **F** The least squares intercept estimator is not affected by changes in units of measure.

T **F** The least squares slope estimator is not affected by the base of the log transformation.

T **F** The least squares intercept estimator is not affected by the base of the log transformation.

T **F** The t statistic for hypotheses about β is not affected by the base of the log transformation.

2. Consider the following simple returns over three successive months: $R_1 = 0.2471$, $R_2 = 0.1653$, and $R_3 = -0.0075$.

   (a) What was the quarterly return over this period?
   (b) What was the average monthly return?
   (c) What were these rates of return on an annual basis?

   In each case give both the discrete and continuous rates, indicating clearly the relationship between the two. Work out all intermediate values to the full internal accuracy of your calculator.

3. Consider the following simple returns over four successive quarters: $R_1 = 0.0690$, $R_2 = 0.0521$, $R_3 = 0.0444$, and $R_4 = -0.0404$.

   (a) What was the annual rate of return?
   (b) What was the average quarterly rate of return?
   (c) What was the average monthly rate of return?
   (d) What was the average monthly rate of return during the second quarter?

   In each case give both the discrete and continuous rates, indicating clearly the relationship between the two. Work out all intermediate values to the full internal accuracy of your calculator.

4. Real variables are obtained from nominal variables by deflating using a price index. If Y denotes some nominal variable, y the variable measured in real terms, and P the price index defined using a base of 100, then the relationship is

$$y = \frac{Y}{P/100}.$$

   Consider the following values for nominal GNP and the price level, the latter measured by the GNP deflator.

Year	Real GNP	Nominal GNP	GNP Deflator
1909		33.400	29.1
⋮		⋮	⋮
1970		974.126	135.3

Average growth rate:

Answer the following questions on this question paper.

(a) Fill in the values for real GNP in 1909 and 1970.

(b) Calculate the average annual growth rates for all three variables, reporting them by filling in the bottom line of the table. Use *continuous* growth rates. Indicate any relationship between your three growth rates.

(c) Suppose you had been given all the annual values for these series between 1909 and 1970, and had used them to calculate mean log-differences like

$$\frac{1}{n-1}\sum_{t=2}^{n}\Delta\log Y_t.$$

The resulting mean log-differences would be

greater than ☐ equal to ☐ less than ☐

your average growth rates of part (b).

(d) Suppose you had used the 1909 and 1970 values to calculate conventional *discrete* growth rates by taking an appropriate $(n-1)$th root. The resulting growth rates would be

greater than ☐ equal to ☐ less than ☐

those of part (b). They

would ☐ would not ☐

satisfy the relationship that exists between the continuous rates of part (b).

(e) Suppose, as in part (c), that you had been given all the annual values for these series between 1909 and 1970; you use them to calculate year-by-year conventional discrete relative differences like

$$\frac{\Delta Y_t}{Y_{t-1}}.$$

You then average these over the $n-1$ values to obtain mean discrete growth rates. These growth rates would be

the same as ☐ different from ☐

those of part (d). They would be

greater than ☐ equal to ☐ less than ☐

those of part (c). They

would ☐ would not ☐

satisfy the relationship that exists between the continuous rates of part (b).

(f) Suppose that instead you modelled the growth in each of these variables by estimating log-lin regressions of the form

$$Y_t = \alpha + \beta t + \varepsilon_t.$$

The slope coefficient β has the interpretation as the growth rate of the series. The least squares estimates of β would be the same as

☐ the growth rates of (b);
☐ the growth rates of (c);
☐ the growth rates of (d);
☐ the growth rates of (e);
☐ all of the above;
☐ none of the above.

5. (This question assumes the provision of the relevant computer output.)

The accompanying regression output gives the estimation results for three Engel curve specifications:

log-log	$\log q_i = \alpha + \beta \log m_i + \varepsilon_i$
lin-log	$q_i = \alpha + \beta \log m_i + \varepsilon_i$
reciprocal	$q_i = \alpha + \beta(1/m_i) + \varepsilon_i$

Logarithmic transformations are done with natural logs. The mean income level is $\bar{m} = \ldots$. Answer the following questions, stating clearly all your steps in any hypothesis tests, including your choice of significance level.

(a) Consider the log-log model.

i. Is income a statistically significant determinant of the consumption of this good? Perform an F test of this regression model, indicating how the value of the test statistic relates to the relevant t statistic value.

ii. Derive the expression for the income elasticity of demand, presenting the simplest derivation you can think of.

iii. Use the regression output to obtain an estimate of this income elasticity. Does it indicate that this good is income elastic, inelastic, or inferior?

iv. Do the estimation results establish clearly that this good is income inelastic, or might this finding be due to sampling error?

(b) Consider the lin-log model. Derive the expression for the income elasticity of demand, and evaluate it at the mean income level. According to this model, at this income level is the good income elastic, inelastic, or inferior?

(c) Consider the reciprocal model. Derive the expression for the income elasticity of demand, and evaluate it at the mean income level. According to this model, at this income level is the good income elastic, inelastic, or inferior?

(d) Using your choice of either the lin-log or reciprocal models, test whether this good is clearly income inelastic (at the mean level of income). Indicate clearly which model you are using.

6. (This question assumes the provision of the relevant computer output.)

Consider Houthakker's data on household income m and electricity consumption q. Instead of the various logarithmic and reciprocal functional forms we have studied, it would be possible to use a standard linear regression to study this Engel curve relationship:

$$q_i = \alpha + \beta m_i + \varepsilon_i.$$

The estimation results for this equation are presented in the accompanying computer output.

(a) For this functional form, derive the appropriate mathematical expression for the elasticity of electricity consumption with respect to income.

(b) The lowest family income in the sample is $m = £279$, the mean family income is $\bar{m} = £592.71$, and the highest family income is $m = £1422$.

i. Compute fitted values $\hat{q}_i$ for each of these income levels, and the corresponding elasticities $\hat{\eta}$. Summarize these in a table.

ii. The accompanying diagram shows the sample regression line and the data. Indicate in the diagram the three points corresponding to your elasticities.

(c) Does your point estimate constitute compelling evidence that, at the point of variable means, the demand for electricity is income inelastic? Taking the sample means $\bar{m}$ and $\bar{q}$ as numerical constants, test the hypothesis that electricity demand is income elastic against the alternative that it is not.