(a) The mean return on the portfolio is $\bar{R}_{\mathrm{P}t} = 0.0325$. This is related to the mean returns on IBM and Xerox individually according to the same linear combination:

$$\bar{R}_{\mathrm{P}} = (1-a)\bar{R}_1 + a\bar{R}_2 = 0.4 \times 0.0212 + 0.6 \times 0.0400 = 0.0325.$$

This was shown previously in Example 5 of Chapter 3.

(b) The risk premium on the portfolio is generated as $R_{Pt} - R_{mt}$. Estimating the CAPM with this as the dependent variable yields $\hat{\beta}_{\mathrm{P}} = 0.992$.

i. This beta is virtually 1 and so this portfolio is effectively neutral.

ii. Denoting the betas of IBM and Xerox (from Exercises 5.16 and 5.17) by $\hat{\beta}_1 = 0.6756$ and $\hat{\beta}_2 = 1.203$, consider the linear combination

$$(1-a)\hat{\beta}_1 + a\hat{\beta}_2 = 0.4 \times 0.6756 + 0.6 \times 1.203 = 0.992.$$

Thus the beta of the portfolio is the linear combination of the individual betas. (This is the empirical counterpart to the theoretical relationship presented at the end of Section C.6.1, Appendix C, and derived in Exercise C.7.)

Examination Questions

1. Consider the following statistical model:

$$Y_i = \alpha + \beta X_i + \varepsilon_i \qquad \varepsilon_i \sim \text{i.i.d.}(0, \sigma^2).$$

(a) What is the formula for the least squares estimator $\hat{\beta}$? (It is not necessary to derive this.) Is this estimator efficient (in the context of the specified model)? Is it BLUE?

(b) Consider the special case of the above model in which $\alpha = 0$. What is the formula for the least squares estimator $\hat{\beta}$? Is this estimator efficient (in the context of the specified model)? Is it BLUE?

(c) Consider the special case of the above model in which $\beta = 0$. What is the formula for the least squares estimator $\hat{\alpha}$? Is this estimator efficient (in the context of the specified model)? Is it BLUE?

2. Least squares estimation of the simple regression model

$$Y_i = \alpha + \beta X_i + \varepsilon_i, \qquad \varepsilon_i \sim \text{n.i.d.}(0, \sigma^2), \qquad (*)$$

gives rise to an unbiased estimator for σ^2 of the form

$$s^2 = \frac{1}{n-?} \sum (Y_i - \hat{Y}_i)^2.$$

Consider the following cases.

(a) For the general model $(*)$, where least squares yields the estimators $\hat{\alpha}$ and $\hat{\beta}$, present

i. the appropriate expressions for $\hat{\alpha}$ and $\hat{\beta}$;
ii. the appropriate value for '?';
iii. the appropriate definition of $\hat{Y}_i$.

(b) For the special case of $(*)$ in which $\beta = 0$, where least squares yields the estimator $\hat{\alpha}$, present

i. the appropriate expression for $\hat{\alpha}$;
ii. the appropriate value for '?';
iii. the appropriate definition of $\hat{Y}_i$.

(c) For the special case of $(*)$ in which $\alpha = 0$, where least squares yields the estimator $\hat{\beta}$, present

i. the appropriate expression for $\hat{\beta}$;
ii. the appropriate value for '?';
iii. the appropriate definition of $\hat{Y}_i$.