nerlove1.dat 4.35 (a) i. This answer is software-specific.

ii. Had the dependent variable costs been reexpressed in thousands of dollars, this would represent a variable scaling of $w = 1000$. Only the t and F statistics and the R^2 would be invariant to this change.

(b) i. The relationship $R^2 = 0.9073 = (0.9525)^2 = r_{XY}^2$ is satisfied on the output.

ii. The relationship $F = 1399.0 = (37.4)^2 = t^2$ is satisfied on the output. Consequently the reported p-values of 0.000 are the same for both statistics.

(c) A plot of the sample regression line against the data reveals that it fails to capture a slight curvature in the relationship of costs to output.

Examination Questions

1. Indicate by checking the appropriate box which of the following are population quantities and which are sample quantities.

	Population	Sample
X_i	☐	☐
Y_i	☐	☐
μ	☐	☐
ε_i	☐	☐
$\hat{\pi}$	☐	☐
σ^2	☐	☐
$\hat{\sigma}^2$	☐	☐
$\bar{Y}$	☐	☐
s^2	☐	☐
σ_{XY}	☐	☐
s_{XY}	☐	☐
ρ_{XY}	☐	☐
r_{XY}	☐	☐
α	☐	☐
β	☐	☐
$\hat{\alpha}$	☐	☐
$\hat{\beta}$	☐	☐
e_i	☐	☐
$\hat{Y}_i$	☐	☐
SST	☐	☐
SSR	☐	☐
SSE	☐	☐
R^2	☐	☐
t statistic	☐	☐
F statistic	☐	☐

2. Indicate whether or not each of the following statistics is, in general, invariant to changes in units of measure.

	invariant	not invariant
price index	□	□
percentage change	□	□
elasticity	□	□
sample mean	□	□
covariance	□	□
correlation	□	□
regression slope estimator	□	□
regression intercept estimator	□	□
slope t statistic for $\beta = 0$	□	□
intercept t statistic for $\alpha = 0$	□	□

3. Consider two random variables X and Y that are jointly normally distributed. You might think of these two variables as, say, family income and clothing expenditure. The bivariate normal distribution is characterized by five parameters: the means μ_X and μ_Y, the variances σ_X^2 and σ_Y^2, and the correlation ρ_{XY}.

   Consider n drawings from this population yielding the sample observations X_i, Y_i $(i = 1, \ldots, n)$.

   (a) What are the least squares estimators for the population means μ_X and μ_Y? (It is not necessary to derive them.) What estimators—expressed explicitly in terms of the sample data—would you suggest for the other three parameters?

   (b) Suppose we take the regression approach to studying these data. It may be shown that the conditional distribution of Y given X is of the form

   $$\mathrm{N}(\mu_Y - \mu_X \rho_{XY} \frac{\sigma_Y}{\sigma_X} + \rho_{XY} \frac{\sigma_Y}{\sigma_X} X,\ \sigma_Y^2(1 - \rho_{XY}^2)).$$

   Since the mean of this conditional distribution is linear in X, it may be reparameterized as the classical normal linear regression model

   $$Y_i = \alpha + \beta X_i + \varepsilon_i.$$

   i. In terms of this reparameterization, what expression in the conditional distribution corresponds to β in the regression model?

   ii. What is the least squares estimator $\hat{\beta}$? (It is not necessary to derive it.)

   iii. Suppose you were unaware of this least squares estimator for β, but were aware of the relationship between the regression model and the conditional normal distribution, and the estimators you have given in part (a).

      A. Use this knowledge to propose an estimator for β.

      B. Establish any relationship between your proposed estimator and the least squares estimator $\hat{\beta}$.

      C. Is your proposed estimator unbiased? (It is not necessary to derive this rigorously.)

4. For each estimator below, indicate the divisor needed to yield an unbiased estimator of the appropriate model parameter. Check the appropriate box on each line.

		Divisor						
Model	Estimator	0	1	n	$n-1$	$n-2$	$n+1$	$n+2$
$Y_i \sim \text{Bernoulli}(\pi)$	$\hat{\pi} = \sum Y_i/?$	☐	☐	☐	☐	☐	☐	☐
$Y_i \sim \text{N}(\mu, \sigma^2)$	$\hat{\mu} = \sum Y_i/?$	☐	☐	☐	☐	☐	☐	☐
$Y_i \sim \text{N}(\mu, \sigma^2)$	$s^2 = \sum y_i^2/?$	☐	☐	☐	☐	☐	☐	☐
$Y_i = \alpha + \beta X_i + \varepsilon_i, \varepsilon_i \sim \text{N}(0, \sigma^2)$	$s^2 = \sum e_i^2/?$	☐	☐	☐	☐	☐	☐	☐
$X_i, Y_i \sim$ bivariate normal	$s_{XY} = \sum x_i y_i/?$	☐	☐	☐	☐	☐	☐	☐