4. A sample of size $n = 3$, taken with replacement from this population of size 4, means that there are $4^3 = 64$ sample draws. Consequently instead of attempting to evaluate the sampling distribution directly, the easiest way to answer this question is to use our more general results.

   (a) By Result 2.1 on the unbiasedness of the sample mean, the mean of the sampling distribution is

   $$E(\bar{Y}) = \mu = 4.5.$$

   (b) By Result 2.3 on the variance of the sample mean, the variance of the sampling distribution is

   $$V(\bar{Y}) = \frac{\sigma^2}{n} = \frac{1/4}{3} = \frac{1}{12}.$$

   (c) Since the variance $V(\bar{Y}) = 1/12$ that applies when $n = 3$ is less than the value $V(\bar{Y}) = 1/8$ that applies when $n = 2$, the larger sample size is preferred.

Examination Questions

1. Consider a population of three balls numbered 1, 2, and 3.

   (a) Present a formula for the population distribution.

   (b) Show how to find the population mean and variance.

   (c) Consider a sample of size 2 drawn with replacement from this population.

      i. Present the sampling distribution of the sample mean.

      ii. Derive the mean of this sampling distribution. Is the sample mean an unbiased estimator of the population mean in this context?

      iii. Derive the variance of this sampling distribution. Indicate any relationship it bears to the population variance.

      iv. What is the probability of a zero sampling error?

   (d) Consider a sample of size 3 drawn with replacement from this population, and the sample mean computed from it.

      i. What is the mean of the sampling distribution?

      ii. What is the variance of the sampling distribution?

2. Consider a population of three balls numbered -1, 0, and 1.

   (a) Present a formula for the population distribution.

   (b) Obtain the population mean and variance.

   (c) Consider the sample mean based on a sample of size 2 drawn with replacement from this population.

      i. Present the sampling distribution of the sample mean.

      ii. Present the mean of this sampling distribution. Is the sample mean an unbiased estimator of the population mean?

iii. Present the variance of this sampling distribution. Indicate any relationship to the population variance.

3. (This question assumes the provision of the relevant computer output.)

   Below are a histogram and descriptive statistics for monthly returns on Xerox over the period July 1963–June 1968. The returns are continuously compounded; that is, they are of the form $\ln(1 + R_t)$, where R_t is a simple return. Such continuously compounded returns are the log-difference of the stock price P_t,

$$\ln(1 + R_t) = \Delta \ln P_t.$$

   Financial economists believe that stock prices are well described as a *random walk with drift,*

$$\Delta \ln P_t = \beta + \varepsilon_t.$$

   Here β denotes the so-called *drift parameter.*

   Answer the following questions.

   (a) From the descriptive statistics on Xerox that are given, present an estimate of β.

   (b) Perform a test of the hypothesis that the drift is zero. Use a 5% level of significance. State all elements of your test procedure carefully.