

algebraically expand $S(\beta_2) = \sum_{i=1}^N (y_i - \beta_2 x_i)^2$ by squaring the term in parentheses and carrying the summation operator through.)

- (d) ♦ Using calculus, show that the formula for the least squares estimate of β_2 in this model is $b_2 = \sum x_i y_i / \sum x_i^2$. Use this result to compute b_2 and compare this value to the value you obtained geometrically.
- (e) Using the estimate obtained with the formula in (d), plot the fitted (estimated) regression function. On the graph locate the point (\bar{x}, \bar{y}) . What do you observe?
- (f) Using the estimates obtained with the formula in (d), obtain the least squares residuals, $\hat{e}_i = y_i - b_2 x_i$. Find their sum.
- (g) Calculate $\sum x_i \hat{e}_i$.

✓ 2.5 A small business hires a consultant to predict the value of weekly sales of their product if their weekly advertising is increased to \$600 per week. The consultant takes a record of how much the firm spent on advertising per week and the corresponding weekly sales over the past 6 months. The consultant writes "Over the past 6 months the average weekly expenditure on advertising has been \$450 and average weekly sales have been \$7500. Based on the results of a simple linear regression, I predict sales will be \$8500 if \$600 per week is spent on advertising."

- (a) What is the estimated simple regression used by the consultant to make this prediction?
- (b) Sketch a graph of the estimated regression line. Locate the average weekly values on the graph.

✓ 2.6* A soda vendor at Louisiana State University football games observes that more sodas are sold the warmer the temperature at game time. Based on 32 home games covering 5 years, the vendor estimates the relationship between soda sales and temperature to be $\hat{y} = -240 + 6x$, where y = the number of sodas she sells and x = temperature in degrees Fahrenheit,

- (a) Interpret the estimated slope and intercept. Do the estimates make sense? Why or why not?
- (b) On a day when the temperature at game time is forecast to be 80°F, predict how many sodas the vendor will sell.
- (c) Below what temperature are the predicted sales zero?
- (d) Sketch a graph of the estimated regression line.

✓ 2.7. You have the results of a simple linear regression based on state-level data and the District of Columbia, a total of $N = 51$ observations.

- (a) The estimated error variance $\hat{\sigma}^2 = 2.04672$. What is the sum of the squared least squares residuals?
- (b) The estimated variance of b_2 is 0.00098. What is the standard error of b_2 ? What is the value of $\sum (x_i - \bar{x})^2$?
- (c) Suppose the dependent variable y_i = the state's mean income (in thousands of dollars) of males who are 18 years of age or older and x_i the percentage of males 18 years or older who are high school graduates. If $b_2 = 0.18$, interpret this result.
- (d) Suppose $\bar{x} = 69.139$ and $\bar{y} = 15.187$, what is the estimate of the intercept parameter?
- (e) Given the results in (b) and (d), what is $\sum x_i^2$?
- (f) For the State of Arkansas the value of y_i is 12.274 and the value of $x_i = 58.3$. Compute the least squares residual for Arkansas. (Hint: Use the information in parts (c) and (d).)