Chapter 14

Heteroskedasticity

This chapter uses some of the applications from previous chapters to illustrate issues in model discovery. No new applications are introduced.

houthak.dat is used in Section 14.2 to illustrate heteroskedasticity-consistent standard errors, in Section 14.3 to illustrate weighted least squares, and in Section 14.4 to illustrate the Breusch-Pagan test.

mrw.dat is used in Section 14.2 to illustrate heteroskedasticity-robust tests of linear restrictions.

nerlove.dat is used in Section 14.2 to illustrate heteroskedasticity-robust tests of linear restrictions.

mizon57.dat is used in Section 14.2 to illustrate heteroskedasticity-robust tests of nonlinear restrictions.

Marginal productivity conditions are used in Exercise 14.6 in a heteroskedasticity-robust Chow test, to illustrate this solution to the Behrens-Fisher problem.

All the exercises of this chapter except Exercise 14.7 are applied rather than theoretical in nature.

Exercise Solutions

14.1 (a) The OLS estimates and standard errors should be verified. houthak.dat

(b) The answer to this question is software-specific.

14.2 The estimation results for Houthakker's linear-reciprocal model are as follows, with size-corrected heteroskedasticity-consistent standard errors reported. houthak.dat

coefficient	variable	estimate	standard error
β_2	m	1.9173	0.214
β_3	$1/p_1$	752.71	167.9
β_4	p_2	1.7510	33.62
β_5	h	286.52	97.60

(a) Under the reciprocal specification, a downward sloping demand curve implies that β_3 should be *positive*.

$H_0 : \beta_3 \leq 0$

$H_A : \beta_3 > 0$

Test Statistic: $t = \dfrac{\hat{\beta}_3 - \beta_3}{s_{\hat{\beta}_3}} \sim t(n-K)$

Rejection Region: $t > t_a(n-5) \approx 1.645$ for $n = 42$ and, say, $a = 0.05$

Conclusion: $t = \dfrac{\hat{\beta}_2}{s_{\hat{\beta}}} = \dfrac{752.71}{167.9} = 4.483$

Since this value is in the rejection region H_0 is rejected. The data clearly indicate a downward sloping demand curve.

The treatment of heteroskedasticity does not alter the previous conclusions.

(b) Since the OLS coefficient point estimates are unchanged, the income elasticity of demand continues to be $\hat{\eta} = 0.86$.

(c) Since the OLS coefficient point estimates are unchanged, the price elasticity of demand continues to be

$$-\frac{1}{\hat{q}p_1}\hat{\beta}_3 = -\frac{1}{1113.738 \times 0.5}752.71 = -1.3672.$$

(d) Since the OLS coefficient point estimates are unchanged, so too are the income and price elasticities. Evaluated at the sample means of

$\bar{m}$	592.71
$\bar{p}_1$	0.54143
$\bar{p}_2$	7.7476
$\bar{h}$	0.69167,

the predicted value of q is $\hat{q} = 1230.77$. Evaluating the elasticity formulas on this basis yields an income elasticity of

$$\hat{\eta} = \frac{m}{\hat{q}}\hat{\beta}_2 = \frac{592.71}{1230.77}1.9173 = 0.92$$

and a price elasticity of

$$-\frac{1}{\hat{q}p_1}\hat{\beta}_3 = -\frac{1}{1230.77 \times 0.54143}752.71 = -1.13.$$

(e) (This answer takes the approach of part (a) of Exercise 6.12. However the approach of part (b) could also be used.)

The relationship (8.28) between η and β_2, evaluated at the point of variable means, is

$$\eta = \frac{m}{\hat{q}}\beta_2 = \frac{592.71}{1230.77}\beta_2 = 0.4816\beta_2.$$

Therefore any hypothesis about one translates into an equivalent hypothesis about the other; in this case,

$$\begin{array}{ll} H_0: & \eta \geq 1 \Leftrightarrow \beta_2 \geq 2.0765 \\ H_A: & \eta < 1 \Leftrightarrow \beta_2 < 2.0765 \end{array}$$

Thus stated as a restriction on β_2, the hypothesis may be tested in the usual way.

Test Statistic: $t = \dfrac{\hat{\beta}_2 - \beta_2}{s_{\hat{\beta}_2}} \sim t(n-K)$

Rejection Region: $t < -t_a(n-5) \approx -1.645$ for $n = 42$ and, say, $a = 0.05$

Conclusion: $t = \dfrac{\hat{\beta}_2 - 2.0765}{s_{\hat{\beta}}} = \dfrac{1.9173 - 2.0765}{0.214} = -0.744$

Since this value is not in the rejection region, H_0 is not rejected. Although the point estimate suggests electricity is income inelastic, $\hat{\beta}_2$ is not significantly less than one.

The treatment of heteroskedasticity does not alter the previous conclusions.

14.3 The results of Example 2 should be replicated. mrw.dat

14.4 The results of Example 3 should be replicated. nerlove.dat

14.5 The results of Example 4 should be replicated. mizon57.dat

14.6 A heteroskedasticity-robust Chow test can be obtained by implementing the dummy-variable version of the test and testing the joint significance of the dummy terms with a heteroskedasticity-robust Wald statistic. The marginal productivity condition, with dummy variable terms included, is basichem.dat

$$\log(V/L)_i = \alpha + \beta \log w_i + \alpha^* D_i + \beta^* D_i \log w_i + \varepsilon_i.$$

The following table presents the relevant test statistics. The F statistic for the restrictions $\alpha^* = \beta^* = 0$ is as obtained previously in Exercises 10.17 and 10.22 using the data for the Basic Chemicals industry. Its p-value is calculated from the $F(2, n-K)$ distribution, where $n = 16$ and $K = 4$. The adjusted Wald statistic is obtained from the unadjusted one by applying the factor $(n-K)/n$. Their p-values are obtained from the $\chi^2(2)$ distribution.

Test	statistic	p-value
F	4.839	0.029
Heteroskedasticity-robust tests:		
Wald (unadjusted)	13.104	0.0014
Wald (adjusted)	9.828	0.0073

The heteroskedasticity-robust tests provide even stronger evidence rejecting the hypothesis of a common marginal productivity condition for all countries than did the F test. Whereas the F test rejected at 5% but not at 1%, the heteroskedasticity-robust tests reject even at 1%.

14.7 The transformed disturbance ε_i^* is defined as

$$\varepsilon_i^* = \frac{\varepsilon_i}{\sqrt{h(\cdot)}},$$

where $h(\cdot)$ is a specified function of known constants and variables and ε_i is a heteroskedastic disturbance where the heteroskedasticity of the form

$$V(\varepsilon_i) = \sigma_i^2 = \sigma^2 h_i(\cdot).$$

The variance of ε_i^* is

$$V(\varepsilon_i^*) = \left(\frac{1}{\sqrt{h(\cdot)}}\right) V(\varepsilon_i) = \frac{1}{h(\cdot)}\sigma^2 h_i(\cdot) = \sigma^2.$$

This establishes that the transformed disturbance ε_i^* is homoskedastic.

14.8 The predicted value for electricity consumption, based on the specified values for the regressors, is

$$\hat{q} = -1700 + 2.378(500) + 609.2\frac{1}{0.5} + 41.58(8.0) + 270.1(0.5) = 1175.09.$$

(a) Evaluating the income elasticity at this point yields

$$\hat{\eta} = \frac{m}{\hat{q}}\hat{\beta}_2 = \frac{500}{1175.09}2.378 = 1.01.$$

This verifies Houthakker's estimate.

(b) For this reciprocal model the price elasticity of demand is

$$\frac{p_1}{q}\frac{\partial q}{\partial p_1} = \frac{p_1}{q}\beta_3(-1)p_1^{-2} = -\frac{1}{qp_1}\beta_3.$$

Evaluating this at $\hat{q} = 1175.09$, $p_1 = 0.5$, and $\hat{\beta}_3 = 609.2$ yields an estimated price elasticity of

$$-\frac{1}{\hat{q}p_1}\hat{\beta}_3 = -\frac{1}{1175.09 \times 0.5}609.2 = -1.04.$$

This verifies Houthakker's estimate.

houthak.dat

14.9 The WLS estimation results for Houthakker's linear-reciprocal model, using the specification $h_i(\cdot) = 1/\sqrt{n_i}$, are as follows.

coefficient	variable	estimate	standard error
β_1	intercept	-1666.6	310.4
β_2	m	2.341	0.201
β_3	$1/p_1$	604.0	124.9
β_4	p_2	40.89	21.17
β_5	h	267.7	61.91

These are similar to those reported by Houthakker (surprisingly so, in view of the computer technology available to him).

(a) Under the reciprocal specification, a downward sloping demand curve implies that β_3 should be *positive.*

$H_0 : \beta_3 \leq 0$

$H_A : \beta_3 > 0$

Test Statistic: $t = \dfrac{\hat{\beta}_3 - \beta_3}{s_{\hat{\beta}_3}} \sim t(n-K)$

Rejection Region: $t > t_a(n-5) \approx 1.645$ for $n = 42$ and, say, $a = 0.05$

Conclusion: $t = \dfrac{\hat{\beta}_2}{s_{\hat{\beta}}} = \dfrac{604.0}{124.9} = 4.836$

Since this value is in the rejection region H_0 is rejected. The data clearly indicate a downward sloping demand curve.

(b) The predicted value for electricity consumption, based on the specified values for the regressors, is

$$\hat{q} = -1666.6 + 2.341(500) + 604.0\frac{1}{0.5} + 40.89(8.0) + 267.7(0.5) = 1172.87.$$

Evaluating the income elasticity at this point yields

$$\hat{\eta} = \frac{m}{\hat{q}}\hat{\beta}_2 = \frac{500}{1172.87}2.341 = 1.00.$$

(c) Evaluating the appropriate elasticity expression at $\hat{q} = 1172.87$, $p_1 = 0.5$, and $\hat{\beta}_3 = 609.2$ yields a price elasticity of

$$-\frac{1}{\hat{q}p_1}\hat{\beta}_3 = -\frac{1}{1175.09 \times 0.5}604.0 = -1.03.$$

14.10 The FWLS estimation results for Houthakker's linear-reciprocal model are as follows. houthak.dat

coefficient	variable	estimate	standard error
β_1	intercept	-1257.9	387.6
β_2	m	2.073	0.230
β_3	$1/p_1$	569.7	143.7
β_4	p_2	6.049	27.74
β_5	h	254.4	73.70

(a) Under the reciprocal specification, a downward sloping demand curve implies that β_3 should be *positive.*

$H_0 : \beta_3 \leq 0$

$H_A : \beta_3 > 0$

Test Statistic: $t = \dfrac{\hat{\beta}_3 - \beta_3}{s_{\hat{\beta}_3}} \sim t(n-K)$

Rejection Region: $t > t_a(n-5) \approx 1.645$ for $n = 42$ and, say, $a = 0.05$

Conclusion: $t = \frac{\hat{\beta}_2}{s_{\hat{\beta}}} = \frac{569.7}{143.7} = 3.965$

Since this value is in the rejection region H_0 is rejected. The data clearly indicate a downward sloping demand curve.

(b) The predicted value for electricity consumption, based on the specified values for the regressors, is

$$\hat{q} = -1257.9 + 2.073(500) + 569.7\frac{1}{0.5} + 6.049(8.0) + 254.4(0.5)$$
$$= 1093.6.$$

Evaluating the income elasticity at this point yields

$$\hat{\eta} = \frac{m}{\hat{q}}\hat{\beta}_2 = \frac{500}{1093.6}2.073 = 0.95.$$

(c) Evaluating the appropriate elasticity expression at $\hat{q} = 1093.6$, $p_1 = 0.5$, and $\hat{\beta}_3 = 569.7$ yields a price elasticity of

$$-\frac{1}{\hat{q}p_1}\hat{\beta}_3 = -\frac{1}{1093.6 \times 0.5}569.7 = -1.04.$$

houthak.dat 14.11 The results are summarized as follows.

estimator	own-price elasticity	own-price t stat ($\beta_3 = 0$)	income elasticity $\hat{\eta}$
(a) OLS	−1.37	4.56	0.86
(b) OLS/White	−1.37	4.48	0.86
(c) Houthakker (1951)	−1.04	4.92	1.01
(d) WLS	−1.03	4.84	1.00
(e) FWLS	−1.04	3.97	0.95

houthak.dat 14.12 The Breusch-Pagan test of Example 6 should be replicated.