Business, Law, and Economics
Professor Richard Boylan

Calculus notes

If f is a function of x, the derivative of f with respect to x is denoted by $f'(x)$ or $\frac{d}{dx}f(x)$. In the following formulas, a, b, c, and n do not depend on x.

Formulas

- Rule 1. If $f(x) = c$, $f'(x) = 0$.
- Rule 2. If $f(x) = ax + b$, $f'(x) = a$.
- Rule 3. If $f(x) = x^n$, $f'(x) = nx^{n-1}$.
- Rule 4. If $g(x) = cf(x)$, $g'(x) = cf'(x)$.
- Rule 5. If $h(x) = f(x) + g(x)$, $h'(x) = f'(x) + g'(x)$.

Problems

Compute the derivative of each of these expressions with respect to x.

1. $\frac{d}{dx}5$.
2. $\frac{d}{dx}6x$.
3. $\frac{d}{dx}6x + 4$.
4. $\frac{d}{dx}\frac{1}{x}$.
5. $\frac{d}{dx}\frac{c}{x}$.
6. $\frac{d}{dx}(4 + \frac{5}{x})$.
7. $\frac{d}{dx}(x^2 - 5x)$.

Solutions

1. Here we have a function $f(x) = 5$. So, we can use Rule 1 (where $c = 5$). This gives $f'(x) = \frac{d}{dx}5 = 0$.

2. Here we have a function $f(x) = 6x$. So we can use Rule 2 (where $a = 6$ and $b = 0$). This gives $f'(x) = \frac{d}{dx}6x = 6$.

3. Here we have a function $f(x) = 6x + 4$. So we can use Rule 2 (where $a = 6$ and $b = 4$). This gives $f'(x) = \frac{d}{dx}6x + 4 = 6$.

4. Here we have a function $f(x) = \frac{1}{x}$. So, we can use Rule 3, with $n = -1$. This gives $f'(x) = \frac{d}{dx}\frac{1}{x} = -\frac{1}{x^2}$.

5. Here we have a function $g(x) = cf(x)$, where $f(x) = \frac{1}{x}$. Using Rule 4, we find that $g'(x) = cf'(x)$. In Problem 4, we found that $f'(x) = -\frac{1}{x^2}$. Therefore, we conclude that $g'(x) = \frac{d}{dx}\frac{c}{x} = -\frac{c}{x^2}$.

6. Here we have a function $h(x) = f(x) + g(x)$, where $f(x) = 4$ and $g(x) = \frac{5}{x}$. Using the same method as Problem 1 we find that $f'(x) = 0$. Using the same method as Problem 5 we find that $g'(x) = -\frac{5}{x^2}$. Using Rule 5, we conclude that $h'(x) = \frac{d}{dx}(4 + \frac{5}{x}) = -\frac{5}{x^2}$.

7. Here we have a function $h(x) = f(x) + g(x)$ where $f(x) = x^2$ and $g(x) = -5x$. Using Rule 3, we obtain $f'(x) = 2x$. Using the same method as in Problem 2, we find that $g'(x) = -5$. Using Rule 5, we conclude that $h'(x) = \frac{d}{dx}(x^2 - 5x) = 2x - 5$