Uncertainty

ECON 370: Microeconomic Theory

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Stanley Gilbert

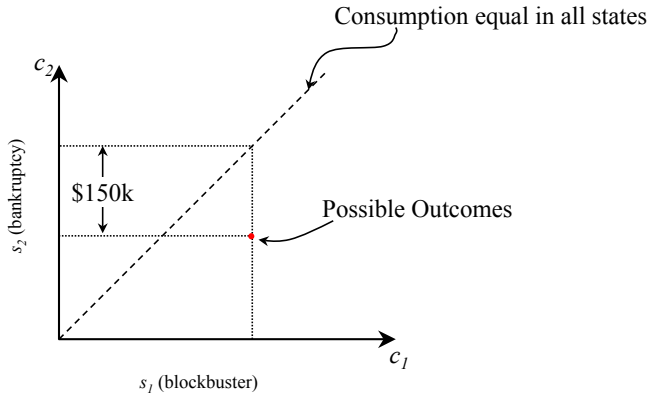
Example

- You own stock in a small pharmaceutical research firm
- There is a chance that next year the drug they are developing will become a blockbuster
  - If they do, your stock will be worth $150,000
- If they drug does not come through they will go bankrupt
- You believe there is a 50% chance that the drug will become a blockbuster
- How do we analyze such situations?

States of the world

- The different possibilities represent different possible states of the world that could obtain in the future
  - Think of them as alternate possible realities
- State s_1, in this case, represents the "blockbuster" case
  - c_1 represents consumption in that state
- State s_2 represents the bankruptcy case
  - c_2 represents consumption in that state
- Represent graphically as follows

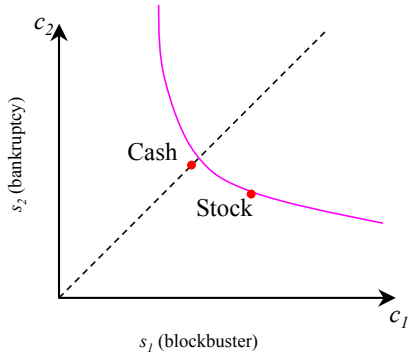
States of the world



Preferences

- We assume people have preferences over consumption in the different states
- So, we can define a utility function like:
  - $U(s_1, c_1, s_2, c_2)$
  - Note that in our previous graph, s_1 and s_2 were invariant
  - The only thing that changed was c_1 and c_2
- In general, we assume people can (to some extent) choose consumption levels across states
  - For example, you can sell your stock for $50,000 cash
  - Which we assume will have the same value in all states

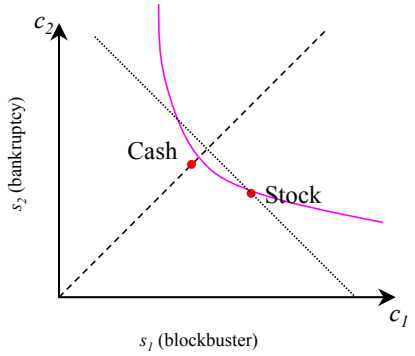
Choices



"Fair-Odds" Line

- Let π_i represent the probability that state i will obtain
  - We write $\pi_i = P\{ s_i \}$
- Assume there are only two possible states.
  - Then $\pi_1 + \pi_2 = 1$
  - Represent consumption for the choice we are analyzing by (c_1^0, c_2^0).
- Then the "fair-odds" line is defined as
  - $\pi_1(c_1 - c_1^0) + \pi_2(c_2 - c_2^0) = 0$
  - Which is a straight line through the consumption bundle and its expected value
  - With a slope equal to the odds ratio

"Fair-Odds" Line



Expected Value and Variance

- Expected Value is defined as:

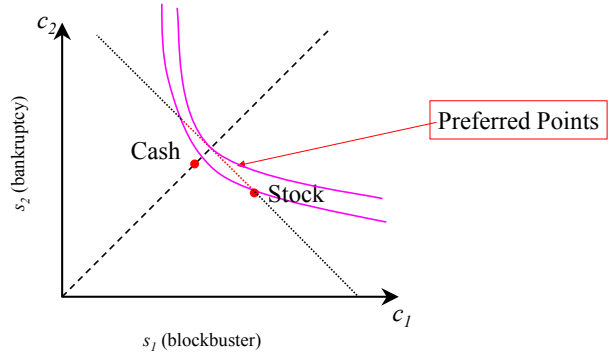
$$EV = \sum_i \pi_i c_i$$

  - It is effectively the average of consumption across states of the world, weighted by likelihood of occurrence
- Variance is: $Var = \sum_i \pi_i (c_i - EV)^2$
- For those more familiar with standard deviation, it is: $SD = \sqrt{Var}$
- Note that the Fair-Odds line crosses the equal-consumption line where $c_1 = c_2 = \text{EV}$

Attitudes toward risk

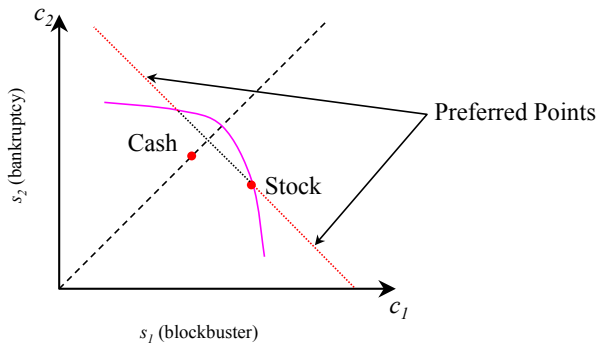
- We say someone is **risk averse** if he prefers points *along the fair-odds line* closer to the center line
  - Alternatively, a risk-averse person, facing several choices with the same Expected Value will choose the one with the lower Variance.
- someone is **risk-loving** if he prefers points *along the fair-odds line* further from the center line
- someone is **risk-neutral** if he is indifferent to all points on the fair-odds line
  - Alternatively, a risk-neutral person, is indifferent to all choices with the same Expected Value.

Risk-Averse

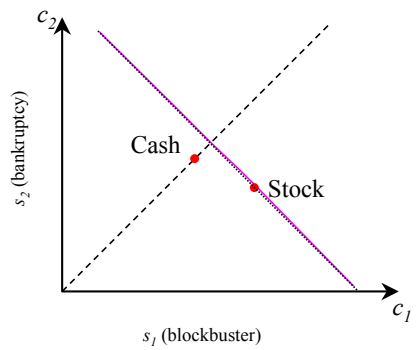


- Note that the most preferred point is where the fair-odds line crosses the equal-consumption line

Risk-Loving



Risk-Neutral



More on Choices

- Most choices have more than two possible outcomes
  - In our example, there may be a third possible state in which pans out, but competition reduces profits so the stock is worth $75,000
- Usually we are choosing between multiple uncertain choices
  - In our example, you may also have the option of investing in a mutual fund that, depending on the state of the economy, will pay somewhere between $40,000 and $75,000 next year

Assumptions about Choices

- We usually make our usual assumptions about choices of uncertain bundles
  - Complete
  - Transitive
  - Monotonicity
- These give us preferences over uncertain choices much like we have been analyzing all along
- If we make a couple of additional assumptions we can represent the utility function in a particularly useful way

(Subjective) Expected Utility

- Given a set of basic assumptions
- We can model people's choices over uncertain bundles as if they were made in accordance to a utility function of the form: $U(s_1, c_1, \cdots, s_n, c_n) = P\{s_1\}u(c_1) + \cdots + P\{s_n\}u(c_n)$
- Where $P\{s_i\}$ represents the probability with which the person believes event s_i will occur
- We usually simplify this to:

$$U(s_1, c_1, \cdots, s_n, c_n) = \pi_1 u(c_1) + \cdots + \pi_n u(c_n)$$

Uniqueness

- The "utility of consumption" function, u(c), is unique only up to a linear transformation
- That is any function:
- $v(c) = a + bu(c)$
- will produce the exact same set of preferences

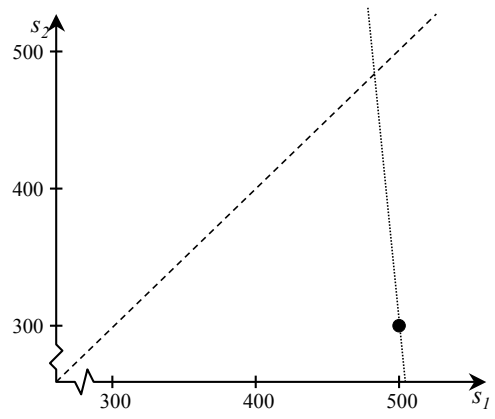
SEU and Attitudes toward risk

- Subjective Expected Utility reflects the attitudes toward risk in a useful way
- Risk-Averse people will have a "utility of consumption" function $u(c)$ that is concave
- Risk-Neutral people will have a "utility of consumption" function $u(c)$ that is linear
- Risk-loving people will have a "utility of consumption" function $u(c)$ that is convex

Example- Insurance

- Assume our consumer owns a house
- The house is worth $200,000
- Total wealth: $500,000
  - We treat total wealth as meaning "I will have $500,000 worth of consumption at some point in time"
- There is a 10% chance the house will burn down
- If it burns down, its value will be $0

Initial Circumstances



Insurance Contract

- A company offers an insurance contract that has the following form:
  - If you pay me $\$x$ today,
  - I will pay you $\$kx$ if state s_2 obtains tomorrow
  - That is, if your house burns down
- Our homeowner must decide:
  - Whether to buy insurance
  - How much to buy

Budget Line

- Since the only means our homeowner has to transfer money from one state to the other is by buying insurance…
- The insurance policy forms our budget constraint
- If he buys $x insurance,
  - Consumption in state s_1 will be $c_1^1 = \$500{,}000 - \$x$
  - Consumption in state s_2 will be
    $c_2^1 = \$300{,}000 - \$x + \$kx$
    $c_2^1 = \$300{,}000 + \$(k-1)x$
- Substituting for c_1^1 for $\$x$ gives
  - $c_2^1 = \$300{,}000 + (k-1)(\$500{,}000 - c_1^1)$

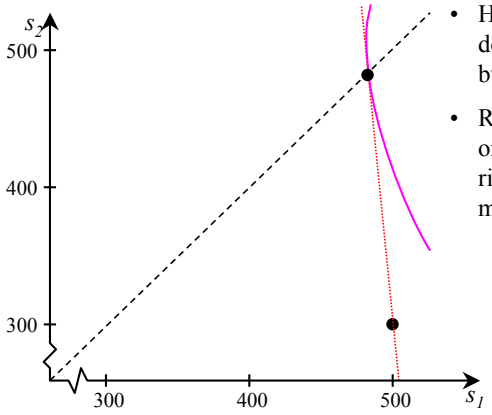
Budget Line (Continued)

- We had
  - $c_2^1 = \$300{,}000 + (k-1)(\$500{,}000 - c_1^1)$
- Rearranging gives us
  - $(k-1)c_1^1 + c_2^1 = \$300{,}000 + \$500{,}000(k-1)$
- If we define k = 1 / γ, then this modifies to
  - $(1-\gamma)c_1^1 + \gamma c_2^1 = \$300{,}000\gamma + \$500{,}000(1-\gamma)$
- If we let π represent the probability of the fire, we can rewrite this as
  - $(1-\gamma)c_1^1 + \gamma c_2^1 = EV - \$200{,}000(\gamma - \pi)$

"Fair" Insurance

- "Fair" Insurance is defined as a policy that moves you along the "fair-odds" line
- Alternatively, "Fair" Insurance does not change the expected value of the outcomes—just the distribution of consumption levels across states
- That is, $\gamma = \pi$
- And the budget line is
  - $(1-\pi)c_1^1 + \pi c_2^1 = EV$

Insurance Choice

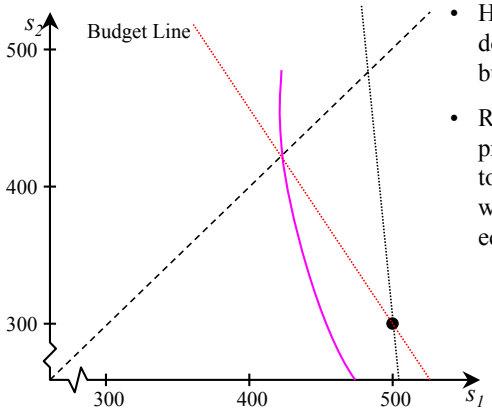


- How much insurance does our homeowner buy?
- Remember what point on the fair-odds line a risk-averse decision-maker will choose.

"Unfair" Insurance

- Insurance companies must meet costs above the amounts needed to pay claims
- So Insurance generally cannot be "fair"
- In our case that means $\gamma > \pi$
- How will our homeowner react to "unfair" insurance?

Insurance Choice



- How much insurance does our homeowner buy?
- Remember that preferences are tangent to the fair-odds line where they pass through equal-consumption line

"Unfair" Insurance

- A Risk-averse homeowner will not fully insure if insurance is "unfair"
- Depending on preferences and the degree of "unfairness" he may not insure at all.