Separating Income and Substitution Effects

ECON 370: Microeconomic Theory

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Effects of a Price Decrease

- Can be broken down into two components
  - Income effect
    - When the price of one goods falls, w/ other constant;
    - Effectively like increase in consumer's real income
    - Since it unambiguously expands the budget set
    - Income effect on demand is positive, *if* normal good
  - Substitution effect
    - Measures the effect of the change in the price *ratio*;
    - Holding some measure of 'income' or well being constant
    - Consumers substitute it for other now relatively more expensive commodities
    - That is, Substitution effect is *always* negative
  - Two decompositions: Hicks, Slutsky

Hicks and Slutsky Decompositions

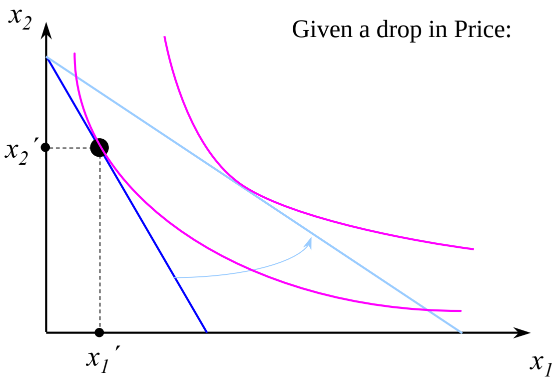
- Hicks
  - *Substitution Effect*: change in demand, holding utility constant
  - *Income Effect*: Remaining change in demand, due to *m* change
- Slutsky
  - *Substitution Effect*: change in demand, holding real income constant
  - *Income Effect*: Remaining change in demand, due to *m* change

Hicks Substitution and Income Effects

- Due to Sir John Hicks (1904-1989; Nobel 1972)
  - To get *Substitution Effect*: Hold utility constant and find bundle that reflects new price ratio
  - Substitution Effect = change in demand due only to this change in price ratio (movement along IC)
  - Income Effect = remaining change in demand to get back to new budget constraint (parallel shift)

Hicks Decomposition Graphically

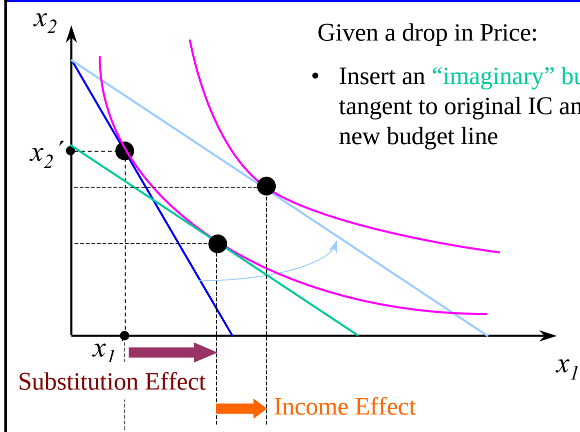
Given a drop in Price:



Hicks Decomposition Graphically 2

Given a drop in Price:

- Insert an "imaginary" budget line tangent to original IC and parallel to new budget line

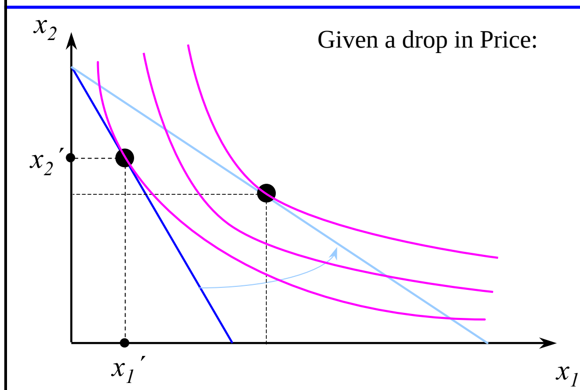


Slutsky Substitution and Income Effects

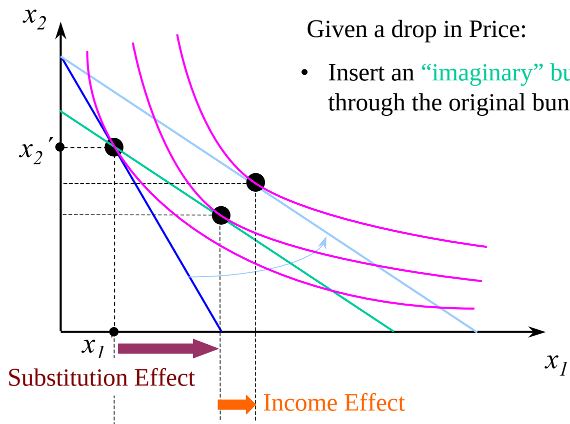
- Due to Eugene Slutsky (1880-1948)
  - To get *Substitution Effect*: Hold purchasing power constant
    - (that is, adjust income so that the consumer can exactly afford the original bundle)
  - and find bundle that reflects new price ratio
  - Substitution Effect = change in demand due only to this change in price ratio (movement along IC)
  - Income Effect = remaining change in demand to get back to new budget constraint (parallel shift)

Slutsky Decomposition Graphically

Given a drop in Price:



Slutsky Decomposition Graphically 2



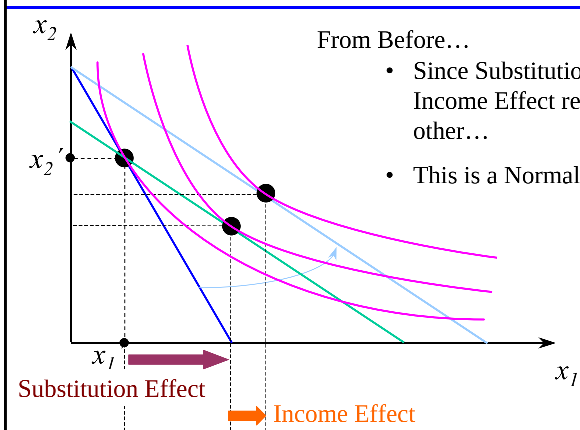
Given a drop in Price:

- Insert an "imaginary" budget line through the original bundle…

Signs of Substitution and Income Effects

- Sign of Substitution Effect is unambiguously negative as long as Indifference Curves are convex
- Income effect may be positive or negative
  - That is, the good may be either *normal* or *inferior*
- For Normal goods, the income effect *reinforces* the substitution effect
- For Inferior goods, the two effects *offset*
- For *Giffen Goods*
  - Remember, the Income effect is Negative
  - **And** the income effect is greater than the substitution effect

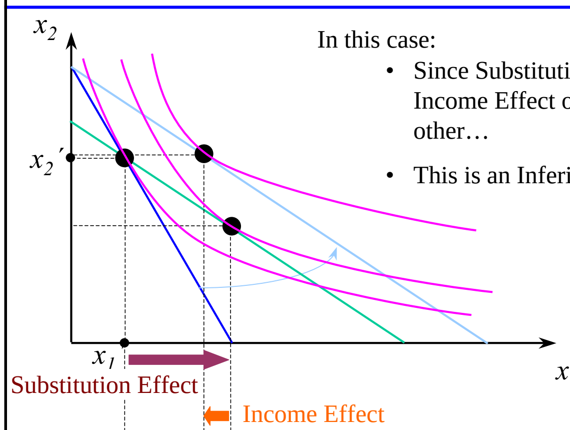
Slutsky's Effects for Normal Goods



From Before…

- Since Substitution Effect and Income Effect reinforce each other…
- This is a Normal Good

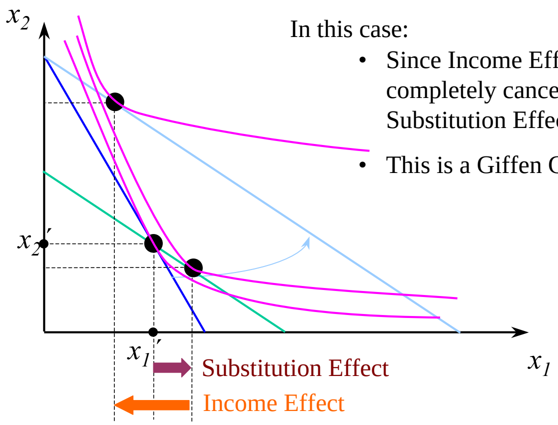
Slutsky's Effects for Inferior Goods



In this case:

- Since Substitution Effect and Income Effect offset each other…
- This is an Inferior Good

Slutsky's Effects for Giffen Goods



In this case:

- Since Income Effect completely cancels the Substitution Effect
- This is a Giffen Good

Mathematics of Slutsky Decomposition

- We seek a way to calculate mathematically the Income and Substitution Effects
- Assume:
  - Income: m
  - Initial prices: p_1^0, p_2
  - Final prices: p_1^1, p_2
  - Note that the price of good two, here, does not change
- Given the demand functions, demands can be readily calculated as:
  - Initial demands: $x_i^0 = x_i(p_1^0, p_2, m)$
  - Final demands: $x_i^1 = x_i(p_1^1, p_2, m)$

Slutsky Mathematics (cont)

- We need to calculate an intermediate demand that holds buying power constant
- Let m^s the income that provides exactly the same buying power as before at the new price
  - Thus: $m^s = p_1^1 x_1^0 + p_2 x_2^0$
- The demand associated with this income is:
  - $x_i^s = x_i(p_1^1, p_2, m^s) = x_i^s(p_1^1, p_2, x_1^0, x_2^0)$
- Finally we have:
  - Substitution Effect: $SE = x_i^s - x_i^0$
  - Income Effect: $IE = x_i^1 - x_i^s$

Hicks' Mathematics

- The only difference is between Hicks' and Slutsky is in the calculation of the intermediate demand
- Let m^h the income that provides exactly the same utility as before at the new price
  - If u^0 is initial utility level, then
  - Thus: m^h solves $u^0 = u(x_1(p_1^1, p_2, m^h), x_2(p_1^1, p_2, m^h))$
- The demand associated with this income is:
  - $x_i^h = x_i(p_1^1, p_2, m^h) = x_i^h(p_1^1, p_2, u^0)$
- Finally we have:
  - Substitution Effect: $SE = x_i^h - x_i^0$
  - Income Effect: $IE = x_i^1 - x_i^h$

Calculating the Slutsky Decomposition

Assume that $u = x^{\alpha} y^{1-\alpha}$

So the demand functions are: $x = \alpha \frac{m}{p_x}$ $\quad y = (1-\alpha)\frac{m}{p_y}$

Initial Price is p_x^0 Final Price is p_x^1

$$x^0 = \alpha \frac{m}{p_x^0} \qquad x^1 = \alpha \frac{m}{p_x^1} \qquad y^0 = y^1 = y = \alpha \frac{m}{p_y}$$

$$m^s = p_x^1 x^0 + p_y y = p_x^1 \alpha \frac{m}{p_x^0} + p_y (1-\alpha)\frac{m}{p_y} = \left[\alpha \frac{p_x^1}{p_x^0} + (1-\alpha)\right] m$$

Calculating the Slutsky Decomposition 2

Since $m^s = \left[\alpha \frac{p_x^1}{p_x^0} + (1-\alpha)\right] m$

We get:

$$x^s = \alpha \frac{m^s}{p_x^1} = \alpha \frac{m}{p_x^1}\left[\alpha \frac{p_x^1}{p_x^0} + (1-\alpha)\right] = \alpha^2 \frac{m}{p_x^0} + \alpha(1-\alpha)\frac{m}{p_x^1}$$

or $x^s = \alpha x^0 + (1-\alpha) x^1$

Finally, we get:

$$SE = x^s - x^0 = \alpha x^0 + (1-\alpha)x^1 - x^0 = (1-\alpha)\left(x^1 - x^0\right)$$

$$IE = x^1 - x^s = x^1 - \left[\alpha x^0 + (1-\alpha)x^1\right] = \alpha\left(x^1 - x^0\right)$$

Calculating the Hicks Decomposition

We need to calculate m^h, so

Substituting our demand functions back into utility we get:

$$u = x^{\alpha} y^{1-\alpha} = \left(\alpha \frac{m}{p_x}\right)^{\alpha}\left((1-\alpha)\frac{m}{p_y}\right)^{1-\alpha} = \left(\frac{\alpha}{p_x}\right)^{\alpha}\left(\frac{1-\alpha}{p_y}\right)^{1-\alpha} m$$

Then m^h solves: $\left(\frac{\alpha}{p_x^1}\right)^{\alpha}\left(\frac{1-\alpha}{p_y}\right)^{1-\alpha} m^h = \left(\frac{\alpha}{p_x^0}\right)^{\alpha}\left(\frac{1-\alpha}{p_y}\right)^{1-\alpha} m$

or $m^h = \left(\frac{p_x^1}{p_x^0}\right)^{\alpha} m$

Calculating the Hicks Decomposition 2

Since $m^h = \left(\frac{p_x^1}{p_x^0}\right)^{\alpha} m$

We get:

$$x^h = \alpha \frac{m^h}{p_x^1} = \alpha \frac{m}{p_x^1}\left(\frac{p_x^1}{p_x^0}\right)^{\alpha} = \alpha \frac{m}{\left(p_x^0\right)^{\alpha}\left(p_x^1\right)^{1-\alpha}}$$

Finally, we get:

$$SE = x^s - x^0 = x^1\left(\frac{p_x^1}{p_x^0}\right)^{\alpha} - x^0 \qquad IE = x^1 - x^s = x^1 - x^1\left(\frac{p_x^1}{p_x^0}\right)^{\alpha}$$

Demand Curves

- We have already met the *Marshallian* demand curve
  - It was demand as price varies, holding all else constant
- There are two other demand curves that are sometimes used
- Slutsky Demand
  - Change in demand holding purchasing power constant
  - The function $x_i^s = x_i(p_1^{1}, p_2, m^s)$ we just defined
- Hicks Demand
  - Change in demand holding utility constant
  - The function $x_i^h = x_i(p_1^{1}, p_2, m^h)$ we just defined

Demand Curves (cont)

- We mentioned before that with Giffen Goods, the Marshallian demand curve slopes upward
- However,
  - Since the substitution effect is always negative, Then
  - Both the Slutsky and Hicks Demands always slope downward—even with Giffen Goods