Ordinal Utility

ECON 370: Microeconomic Theory

Summer 2004 – Rice University

Stanley Gilbert

Lecture 2

Utility Theory

- For consumer theory do not need to assign precise numerical values to different indifference curves
- Often convenient to do so especially if dealing with more than 2 goods
- To do this, it is convenient to *represent* the preference relation by a ***utility function*** $U(x_1,x_2,\ldots)$

Conditions

- $U(x)$ is a utility function representing the preference relation '$\succsim$':
  - $U(x) \geq U(y)$ if and only if $x \succsim y$
  - For preferences that satisfy *A1 – A4*

Construction

- Construction of a utility function:
  - Draw the indifference curves
  - Drawing the diagonal (i.e. the line $x_1 = x_2$)
  - Labeling each indifference curve with how far from the origin the intersection of that indifference curve and the diagonal it is.

Identical Representations

- Many different utility functions can describe same set of preferences
- Any monotonic transformation of a utility function is still a utility function
- For Example: $U(x) = x_1 x_2$

$$U^a(x) = 4x_1^2 x_2^2 + 42$$

$$U^b(x) = (x_1 x_2)^{1/2}$$

$$U^c(x) = \ln x_1 + \ln x_2$$

Equivalence

- Our model of consumer behavior would be exactly the same whichever of these utility functions were assumed.
- See that:

$$U^a(x) = [U(x)]^2 + 42$$

$$U^b(x) = [U(x)]^{1/2}$$

$$U^c(x) = \ln[U(x)]$$

All three are *monotonic* transformations of $U(x)$.

Characteristics

- Consumer theory depends only on the shapes of indifference curves and the direction in which utility is increasing (normally northeast). Most economists argue that precise numerical values attached to individual indifference curves have no economic meaning
- Because the ordering of indifference curves is all that is important, this is described as an *ordinal theory of utility*
- That is, it can only be used to rank choices for a single individual

Interpersonal Comparisons

- An associated issue is that of *interpersonal comparability*:
  - If Jack consumes cans of soda and pizzas and prefers more of each good to less, we can conclude that he would prefer 3 cans of soda and 2 pizzas per day to 1 can of soda and 1 pizza per day
  - Suppose, however, that we have two consumers Jack and Jill and Jack consumes 3 cans of soda and 2 pizzas per day while Jill consumes 1 can of soda and 1 pizza per day. Can we say Jack is happier?
- There is no objective (i.e. *value-free*) way to compare the happiness of the two people
- In our analysis we treat utility functions as ordinal

Common Utility Functions

- **Perfect Substitutes**:
  - $U(x) = ax_1 + bx_2$ $\quad a, b > 0$
- **Perfect Complements**:
  - $U(x) = \min\{ ax_1, bx_2\}$ $\quad a, b > 0$
- **Cobb-Douglas Preferences**
  - $U(x) = x_1^{\alpha} x_2^{1-\alpha}$ $\quad 0 \leq \alpha \leq 1$
  - $U(x) = \alpha \ln(x_1) + (1 - \alpha)\ln(x_2)$ $\quad 0 \leq \alpha \leq 1$
  - (Why are these equivalent?)
- Quasilinear Preferences
  - $U(x) = x_1 + f(x_2)$

Indifference Curves

- If we have a utility function $u = f(x_1, x_2)$:
- Indifference curves are represented by
  - $c = f(x_1, x_2)$
  - Where 'c' is some constant

Marginal Utility

- Marginal means "incremental".
- MU_i = change in U with small change in x_i (holding constant consumption of all other goods)

$$MU_i = \frac{\partial U}{\partial x_i}$$

- For example, if $\quad U(x_1, x_2) = x_1^{1/2} x_2^2$
- Then, $\quad MU(x_1) = (½)x_1^{-1/2} x_2^2$
- And, $\quad MU(x_2) = 2x_1^{1/2} x_2$

Marginal Utilities and the MRS

- The equation of an indifference curve is $U(x_1, x_2) \equiv k$, a constant.
- Totally differentiating this identity gives

$$\frac{\partial U}{\partial x_1} dx_1 + \frac{\partial U}{\partial x_2} dx_2 = 0 \qquad \text{or}$$

$$MRS = \frac{dx_2}{dx_1} = -\frac{\partial U / \partial x_1}{\partial U / \partial x_2} = -\frac{MU(x_1)}{MU(x_2)}$$

Example of Marginal Utilities and MRS

- Suppose $U(x_1,x_2) = x_1x_2$. Then

$$\frac{\partial U}{\partial x_1} = (1)(x_2) = x_2$$

$$\frac{\partial U}{\partial x_2} = (x_1)(1) = x_1$$

so $$MRS = \frac{dx_2}{dx_1} = -\frac{\partial U/\partial x_1}{\partial U/\partial x_2} = -\frac{x_2}{x_1}$$

Monotonic Transformations and MRS

- Monotonic transformation of *U* does not change underlying preference structure
- Example: $U(x_1, x_2) = x_1x_2$ with MRS = $-x_2/x_1$.
- Consider $V = U^2$, or $V(x_1, x_2) = x_1^2x_2^2$:

$$MRS = -\frac{\partial V/\partial x_1}{\partial V/\partial x_2} = -\frac{2x_1x_2^2}{2x_1^2x_2} = -\frac{x_2}{x_1}$$

- This is the same as the MRS for *U*.
- Monotonic transformation does not change MRS.