# Game Theory

ECON 370: Microeconomic Theory

Summer 2004 – Rice University

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## Game Theory: Introduction

- Game theory
  - A means of modeling strategic behavior
  - Agents act to maximize own welfare
  - Agents understand their actions affect actions of other agents



## Game Theory: Applications

- Game theory has been applied to analyze
  - Oligopolies
  - Cartels: OPEC
  - Tax competition across jurisdictions, countries
  - Military strategies
  - Externalities: using common resources like fishery



## Game Theory: Overview

- A *game* consists of
  - a set of *players* (often two)
  - a set of *strategies* for each player
  - the *payoffs* to each player for all combinations of possible strategy choices by the players
- Simplest possible game
  - Two-player game
  - Each chooses between only two strategies

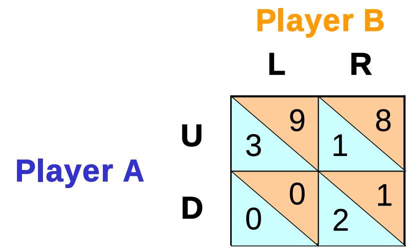
## Two-Player Game: Example

- Players are A and B
- A has two strategies: “Up” and “Down”
- B has two strategies: “Left” and “Right”
- *Payoff matrix* - table showing payoffs to A and B for each of four possible strategy combinations



## Two-Player Game: Matrix

Payoff matrix for game




If A plays ‘D’, what will B do?

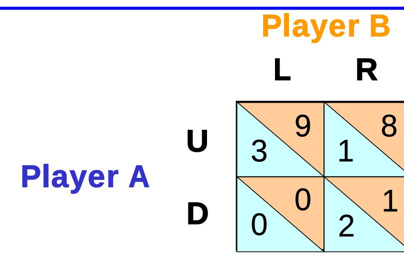


## Nash Equilibrium: Introduction

- *Nash equilibrium*
  - a play where each strategy is a best response to the other
  - Can be multiple Nash equilibria
- Example has two Nash equilibria
  - (U,L) and
  - (D,R)



## Nash Equilibrium: Matrix




- (U,L) and (D,R) are both Nash equilibria
- But (U,L) is preferred to (D,R) by both A & B
- (U,L) is Pareto preferred to (D,R)
- Is (U,L) the only (likely) equilibrium?
  - NO

## Prisoner's Dilemma Game

Here: S=Silence, C=Confess, Payoffs are years in jail

| | | Clyde | |
|---|---|---|---|
| | | S | C |
| Bonnie | S | -5 / -5 | -1 / -30 |
| | C | -30 / -1 | -10 / -10 |

- Only Nash equilibrium for this game is (C,C),
- even though (S,S) gives both players better payoffs
- So, the only Nash equilibrium is inefficient



## Sequential Games

- Thus far, *simultaneous play games*
  - Both players choose strategies simultaneously
- *Sequential play games*
  - One player plays before the other player
  - *Leader* - player who plays first
  - *Follower* - player who plays second
  - May be possible to choose among alternative Nash equilibria



## Sequential Game: Matrix

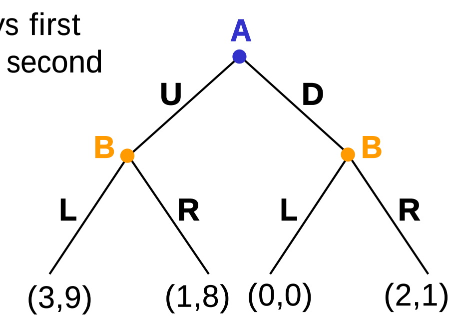
| | | Player B | |
|---|---|---|---|
| | | L | R |
| Player A | U | **9** / **3** | 8 / 1 |
| | D | 0 / 0 | **1** / **2** |

- Simultaneous game: (U,L) and (D,R) are both Nash equilibria
- No obvious way to choose between them



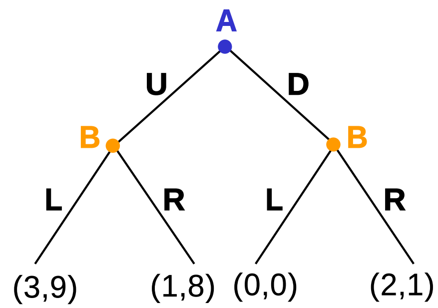
## Sequential Game: Extensive Form

A plays first
B plays second




- Solution proceeds from the end to the beginning
- "What will B do if A chooses 'U'/'D'?"
- Then "What will A do knowing how B will react?"

## Sequential Game: Extensive Form




- If A plays U then B plays L; A gets 3
- If A plays D then B plays R; A gets 2
- A anticipates B, so (U,L) is likely Nash equilibrium



## Mixed Strategies

- Thus far, all "*pure*" strategies
  - Players choose a single strategy
  - E.g., player A plays only U or only D
- Alternative: *Mixed* strategies
  - Choose combination of stratgies
  - Example: A chooses
    - U with probability 0.25 and
    - D with probability 0.75



## Pure Strategies: New Matrix

| | | Player B | |
|---|---|---|---|
| | | L | R |
| Player A | U | 1, 2 | 0, 4 |
| | D | 0, 5 | 3, 2 |

- Do pure strategy Nash equilibria exist?
- No



## Mixed Strategies: Player A

- Suppose Player A chooses *mixed strategy*
  - with probability $\pi_U$ Player A plays Up, and
  - with probability $1-\pi_U$ Player A plays Down
- I.e., mixing pure strategies
- Mixed strategy has probability distribution
- $(\pi_U, 1-\pi_U)$

## Mixed Strategies: Player B

- Similarly, Player B has mixed strategy with probability distribution $(\pi_L, 1-\pi_L)$
  - with probability $\pi_L$ Player B plays Left and
  - with probability $1-\pi_L$ Player B plays Right
- Nash Equilibrium in Mixed Strategies
  - Each player chooses optimal probabilities, given opponent's probabilities
  - Each set of expectations satisfied in eq'm.



## Mixed Strategies

- Assuming players are risk-neutral
- They will pick the alternative with the highest expected value
- If they are randomizing
  - Then they do not clearly prefer one option to another
- That is, Expected Values of both alternatives must be equal
- Assume both players randomize
  - Player A plays alternative U with probability μ
  - Player B plays alternative L with probability λ



## Calculating Mixed Strategies A

**Player B**

| Player A | | L | R |
|---|---|---|---|
| | U | 1, 2 | 0, 4 |
| | D | 0, 5 | 3, 2 |

For player A to be indifferent between U and D

We must have: $1\mu + 0(1-\mu) = 0\mu + 3(1-\mu)$

or $\mu = 0.75$



## Calculating Mixed Strategies B

**Player B**

| Player A | | L | R |
|---|---|---|---|
| | U | 1, 2 | 0, 4 |
| | D | 0, 5 | 3, 2 |

For player B to be indifferent between L and R

We must have: $2\lambda + 5(1-\lambda) = 4\lambda + 2(1-\lambda)$

or $\lambda = 0.6$

## Expected Payoffs

**Player B**

| | | L 0.75 | R 0.25 |
|---|---|---|---|
| **Player A** | U 0.6 | 1, 2 | 0, 4 |
| | D 0.4 | 0, 5 | 3, 2 |

$$Payoff\ \ A = \frac{3}{4}\frac{3}{5}1 + \frac{1}{4}\frac{3}{5}0 + \frac{3}{4}\frac{2}{5}0 + \frac{1}{4}\frac{2}{5}3 = 0.75$$

$$Payoff\ \ B = \frac{3}{4}\frac{3}{5}2 + \frac{1}{4}\frac{3}{5}4 + \frac{3}{4}\frac{2}{5}5 + \frac{1}{4}\frac{2}{5}2 = 3.2$$



## Existence of Nash Equilibrium

- Consider game with
  - finite number of players
  - each with a finite number of pure strategies
- Such a game has
  - at least one (pure or mixed strategy) Nash equilibrium
  - If no pure strategy Nash equilibrium, then must have at least one mixed strategy Nash equilibrium