The Firm

ECON 370: Microeconomic Theory

Summer 2004 – Rice University

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The Firm

- A Firm is a mechanism for converting labor, capital and raw materials into desirable goods
- A firm is owned by consumers and operated for the benefit of its owners (for now)
- We assume that a firm's objective is to maximize profits

Decisions

- Firms have three decisions that they must make:
  - How much to produce
  - How to produce it
  - Whether to produce at all
- We will examine each of these decisions

Maximizing Profits

- Profit is:
  - π = Revenue – Cost
- We generally treat all these as *flows*
  - Profits *per week*
  - Revenue *per week*
  - Costs *per week*
- Maximizing profits implies:
  - Marginal Revenue = Marginal Cost
  - MR = MC
- This is the basic rule that applies to all firms

More on Costs

- We are interested in *Economic* Costs not Accounting Costs
- Economic Cost is the benefit you would get from the best available alternate use of the input
- Examples:
- Sunk expenditures are sunk, they are not costs
  - Iridium
- "Rental" price of inputs
  - Delivery Trucks
  - Owner's time

Production Plans

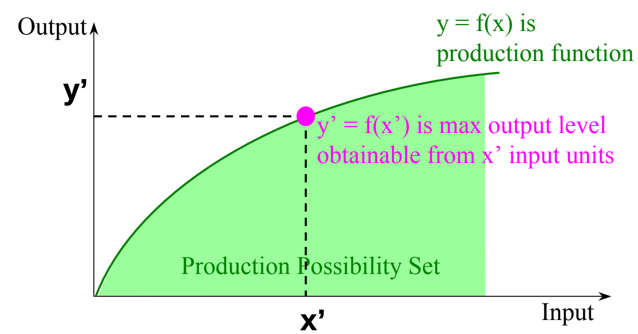
- Firms use some *technology* for transforming inputs into valuable outputs
- With a particular technology, a firm can transform a specific set of inputs (x_1, x_2, …) into not more than some y amount of output
- We represent a technology as:
  - $y = F(x_1, x_2, \ldots)$
  - Where $F(x_1, x_2, \ldots)$ is the *production function* for this technology
- A *production plan* is an input bundle and output level:
  - $(x_1, x_2, \ldots, y)$
- A *feasible* production plan will satisfy
  - $y \leq F(x_1, x_2, \ldots)$

Technology Set

- The *technology set* is the set of all feasible production plans
- That is, all production plans that satisfy
  - $y \leq F(x_1, x_2, \ldots)$
- The technology set is also called the *Production Possibility Set*
- Note that a profit maximizing firm will produce on the Production Possibility Frontier
  - that is, where $y \leq F(x_1, x_2, \ldots)$
  - Why?
  - We call such production plans *Technically Efficient*

Technology Set Graph

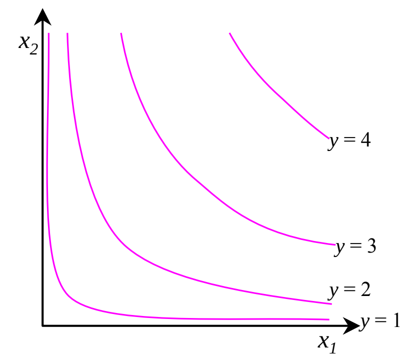
One input and one output



Isoquants

- One way of representing the production function graphically is by drawing *isoquants*
- An isoquant represents all combinations of inputs that will produce a given level of output
- In the two-input case:
  - All x_1, x_2 such that
  - $c = F(x_1, x_2)$

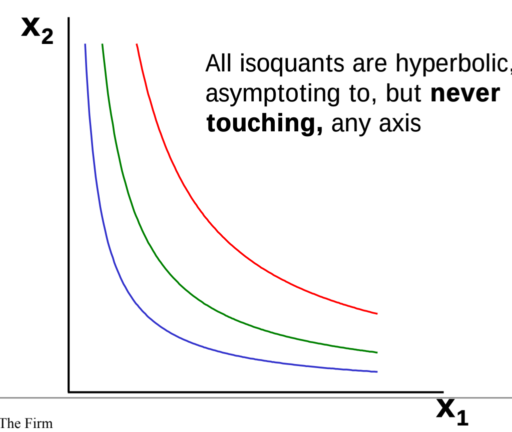
Isoquant Map



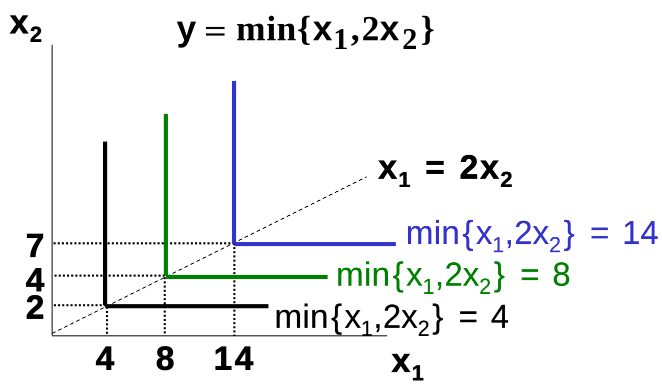
Common Utility Functions

- **Perfect Substitutes**:
  - $F(x) = a_1x_1 + a_2x_2 + \ldots + a_nx_n$ $\quad a_i > 0$
- **Fixed Proportions**:
  - $F(x) = \min\{a_1x_1, a_2x_2, \ldots, a_nx_n\}$ $\quad a_i > 0$
- **Cobb-Douglas Preferences**
  - $F(x) = x_1^{a1}x_2^{a2}\ldots x_n^{an}$ $\quad a_i > 0$

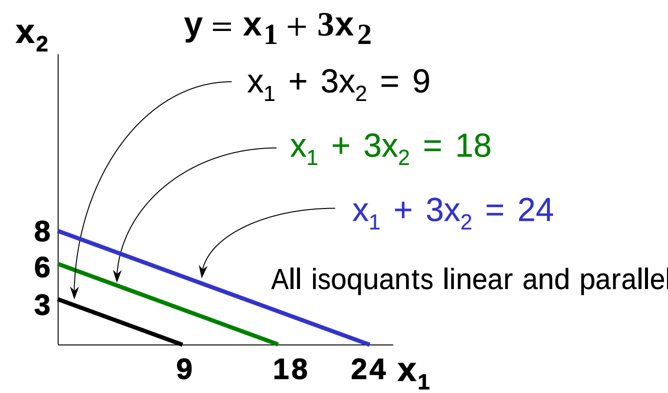
Cobb-Douglas Technologies: Graph



Fixed-Proportions Technologies: Graph



Perfect-Substitution Technologies: Graph



Marginal (Physical) Products

- Given a technology:
  - $y = F(x_1, x_2, \ldots, x_n)$
- Marginal product of input *i*
  - is change in output as input *i* changes,
  - holding all other input levels fixed

$$MP_i = \frac{\partial y}{\partial x_i}$$

- Cobb-Douglas Example
- Typically, marginal product of one input depends on amount used of other inputs

Marginal Products: Diminishing MP

- MP_i is *diminishing* if it declines as x_i increases
- That is, if

$$\frac{\partial MP_i}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\frac{\partial y}{\partial x_i}\right) = \frac{\partial^2 y}{\partial x_i^2} < 0$$

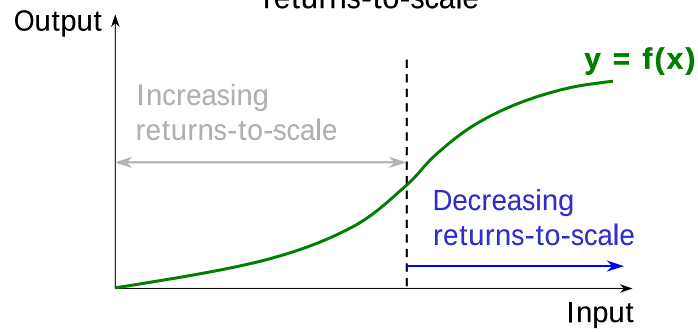
- Cobb-Douglas Example

Returns-to-Scale

- Returns-to-scale
  - Change in output as **all inputs** change proportionally
  - e.g. all input levels doubled, or halved
- Constant Returns to Scale:
  - doubling all inputs, doubles output
  - $F(kx_1, kx_2, \ldots, kx_n) = kF(x_1, x_2, \ldots, x_n)$
- Decreasing Returns to Scale:
  - doubling all inputs, less than doubles output
  - $F(kx_1, kx_2, \ldots, kx_n) < kF(x_1, x_2, \ldots, x_n)$
- Increasing Returns to Scale:
  - doubling all inputs, more than doubles output
  - $F(kx_1, kx_2, \ldots, kx_n) > kF(x_1, x_2, \ldots, x_n)$

Different Returns to Scale (RTS)

A single technology can 'locally' exhibit different returns-to-scale



Examples of RTS: Perfect Substitutes

The perfect-substitutes production function is

$$y = a_1x_1 + a_2x_2 + \cdots + a_nx_n$$

Expand all input levels proportionately by k:

$$\begin{aligned} y' &= a_1(kx_1) + a_2(kx_2) + \cdots + a_n(kx_n) \\ &= k(a_1x_1 + a_2x_2 + \cdots + a_nx_n) \\ &= ky \end{aligned}$$

The perfect-substitutes production function is CRS

Examples of RTS: Perfect Complements

Perfect-complements production function is

$$y = \min\{a_1x_1, a_2x_2, \cdots, a_nx_n\}$$

Expand all input levels proportionately by k:

$$\begin{aligned} y' &= \min\{a_1(kx_1), a_2(kx_2), \cdots, a_n(kx_n)\} \\ &= k(\min\{a_1x_1, a_2x_2, \cdots, a_nx_n\}) \\ &= ky \end{aligned}$$

The perfect-complements production is CRS

Examples of RTS: Cobb-Douglas

The Cobb-Douglas production function is

$$y = x_1^{a_1} x_2^{a_2} \cdots x_n^{a_n}$$

Expand all input levels proportionately by k:

$$\begin{aligned} y' &= (kx_1)^{a_1}(kx_2)^{a_2}\cdots(kx_n)^{a_n} \\ &= k^{a_1}k^{a_2}\cdots k^{a_n} x^{a_1} x^{a_2}\cdots x^{a_n} \\ &= k^{a_1+a_2+\cdots+a_n} x_1^{a_1} x_2^{a_2}\cdots x_n^{a_n} \\ &= k^{a_1+\cdots+a_n} y \end{aligned}$$

Examples of RTS: Cobb-Douglas

$$y' = k^{a_1+\cdots+a_n} y$$

The Cobb-Douglas technology's RTS:

constant if $a_1 + \dots + a_n = 1$

increasing if $a_1 + \dots + a_n > 1$

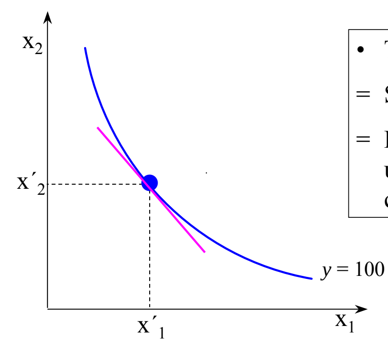
decreasing if $a_1 + \dots + a_n < 1$

Returns-to-Scale and Diminishing MP

- RTS and Marginal Product (MP)
  - RTS refers to change in all inputs
  - MP refers to change in one input, holding all others constant
  - Declining MP reflects each new input having less of others to "work with" and becoming less productive
  - With RTS, each input has same amount of other inputs to "work with" so RTS need not diminish
- Illustration
  - Can have increasing RTS and diminishing MP
  - Cobb-Douglas Production Function

Technical Rate-of-Substitution: Graph

At what rate can a firm substitute one input for another without changing output?



- Technical rate-of-substitution

= Slope of isoquant

= Rate at which x_2 must be given up as x_1 is increased to keep *y* constant

Technical Rate-of-Substitution

- Output y is constant along isoquant
- Production function $y = \mathrm{F}(x_1, x_2)$
- A small change (dx_1, dx_2) in input bundle causes a change to output level y of:

$$dy = \frac{\partial y}{\partial x_1} dx_1 + \frac{\partial y}{\partial x_2} dx_2 = 0$$

$$TRS = \frac{dx_2}{dx_1} = -\frac{\partial y / \partial x_1}{\partial y / \partial x_2} = -\frac{MP_1}{MP_2}$$