General Equilibrium/Exchange

ECON 370: Microeconomic Theory

Summer 2004 – Rice University

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General vs. Partial Equilibrium

- Partial equilibrium
  - Analysis of a single market
  - Assumes price, quantity interactions with all other markets small enough to be ignored
- General equilibrium
  - Considers all interactions among markets
  - Simultaneously obtains prices in all markets
  - Consistent relationships among all markets for all variables (factors, incomes, etc.)

General Equilibrium Analysis

- Start with theory of pure exchange
  - Assume fixed set of consumer goods (ignore how these goods are produced)
  - Assign goods to individuals: endowments
  - Analyze exchange of endowments
- Will consider "production" of these goods later in general equilibrium context
- Will analyze 2-consumer, 2-good model

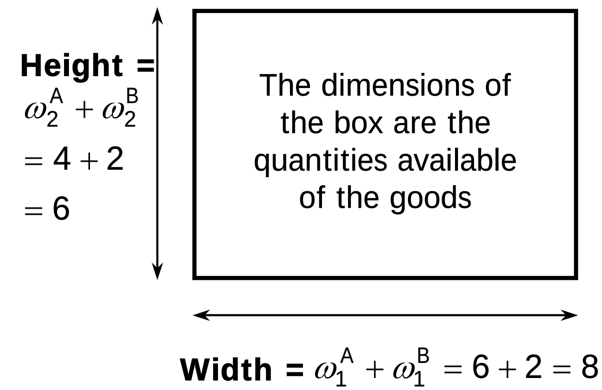
Exchange: Introduction

- Two consumers, A and B
- Endowments of goods 1 and 2 are
- $\omega^A = (\omega_1^A, \omega_2^A)$ for Consumer A
- $\omega^B = (\omega_1^B, \omega_2^B)$ for Consumer B
  - E.g., $\omega^A = (6, 4)$ and $\omega^B = (2, 2)$
- Total quantities available are
  - $\omega_1^A + \omega_1^B = 6 + 2 = 8$ units of good 1
  - $\omega_2^A + \omega_2^B = 4 + 2 = 6$ units of good 2

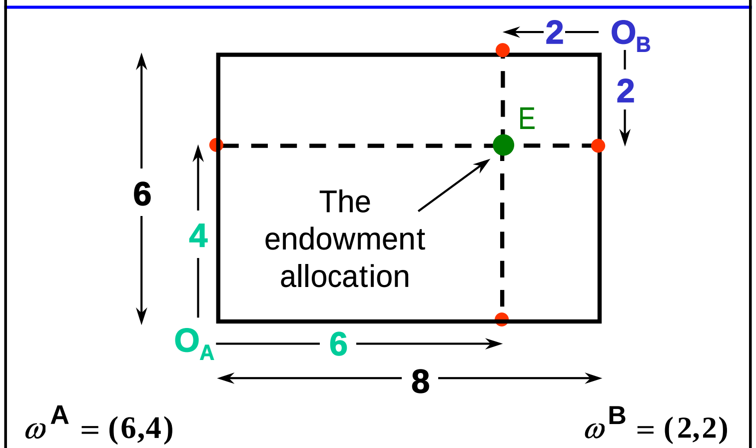
Exchange: The Edgeworth Box

- Theory of Exchange
  - Modeled using *Edgeworth box*
  - Shows all possible *allocations* of fixed quantities of goods 1 and 2 between consumers A and B
  - Start with initial allocation = endowments
  - After trade, end up with final allocation
  - Traded quantities must be consistent

Starting an Edgeworth Box



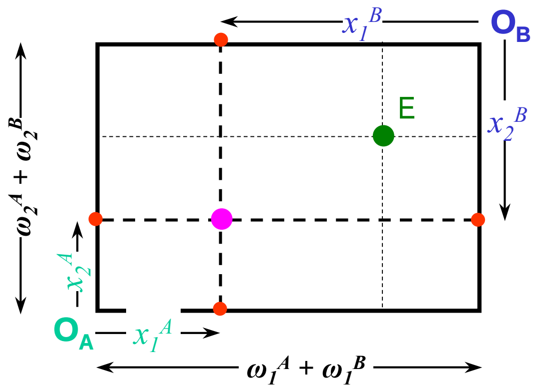
Edgeworth Box: The Endowment



Other Feasible Allocations

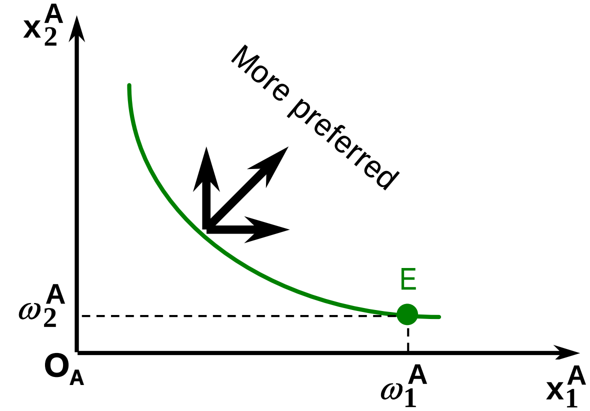
- (x_1^A, x_2^A) denotes an allocation to consumer A
- (x_1^B, x_2^B) denotes an allocation to consumer B
- Feasible allocation satisfies
  - $x_1^A + x_1^B \le \omega_1^A + \omega_1^B$ and
  - $x_2^A + x_2^B \le \omega_2^A + \omega_2^B$
- So, all points in Edgeworth box (including those on boundary) are *Feasible*

Feasible Reallocations: Diagram



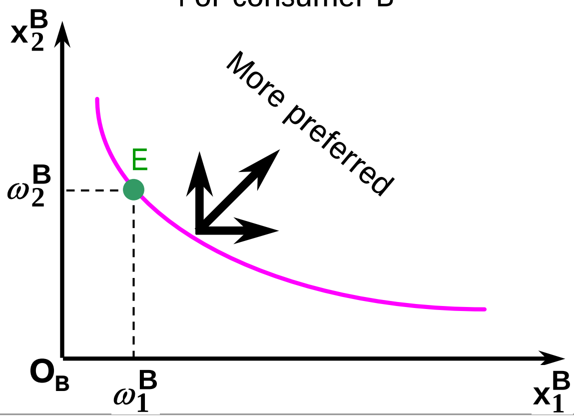
Adding Preferences - Consumer A

For consumer A



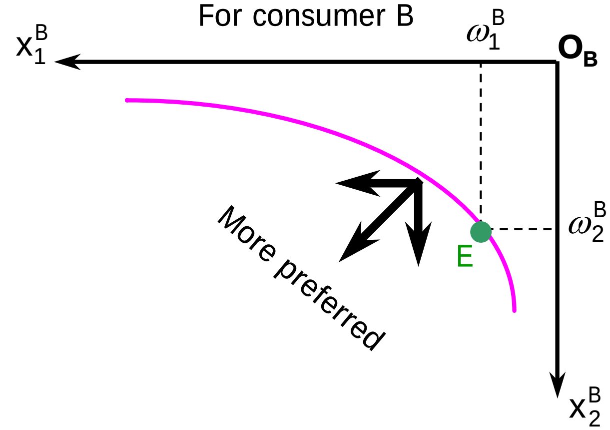
Adding Preferences - Consumer B

For consumer B

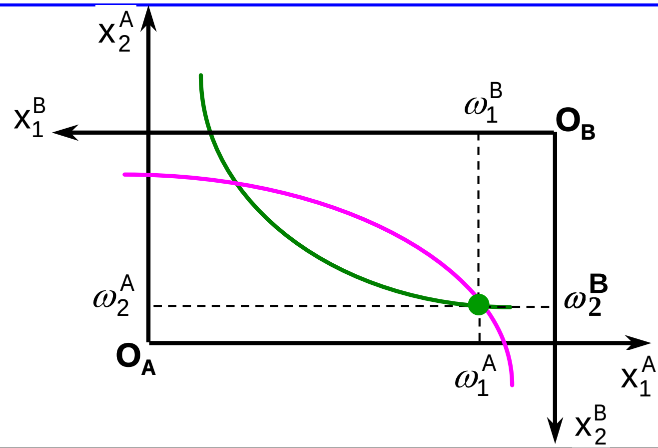


Adding Preferences – Consumer B

For consumer B



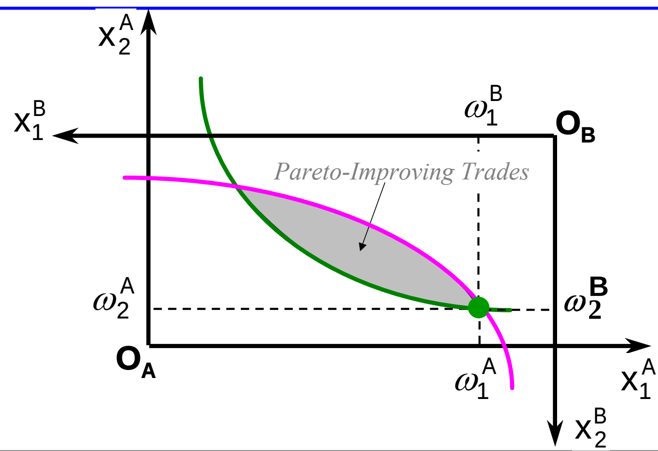
Edgeworth Box



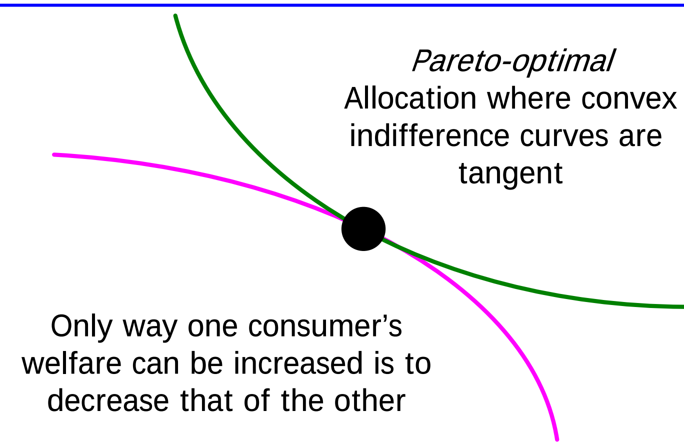
Pareto-Improving Reallocations

- *Pareto-improving reallocation*
  - Re-allocation of endowment improving welfare of one consumer w/o reducing welfare of the other
- Only Pareto-improving reallocations will occur through voluntary trade
- Can be easily identified on Edgeworth box diagram

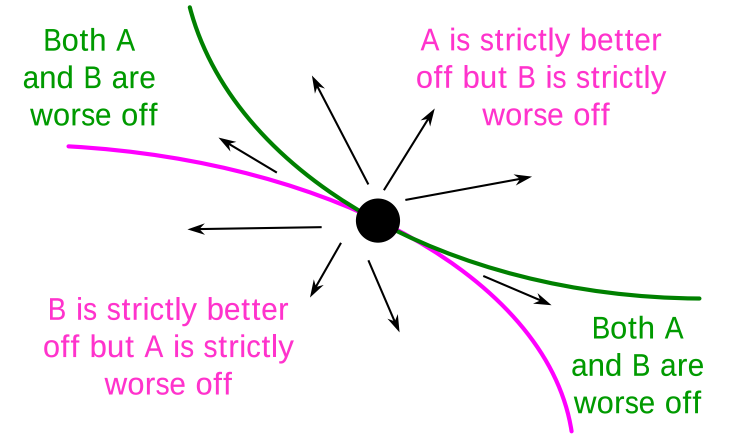
Pareto Improving Trades



Pareto-Optimality: Diagram

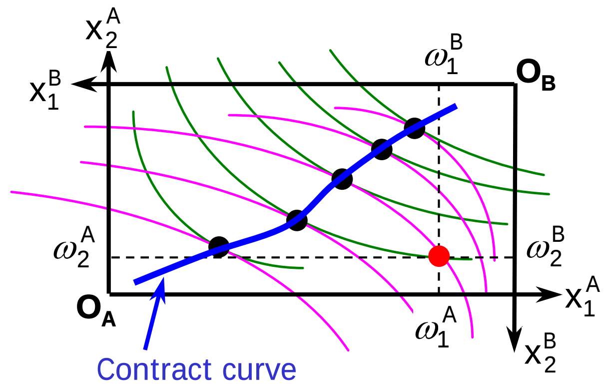


Pareto-Optimality: Diagram



Contract Curve

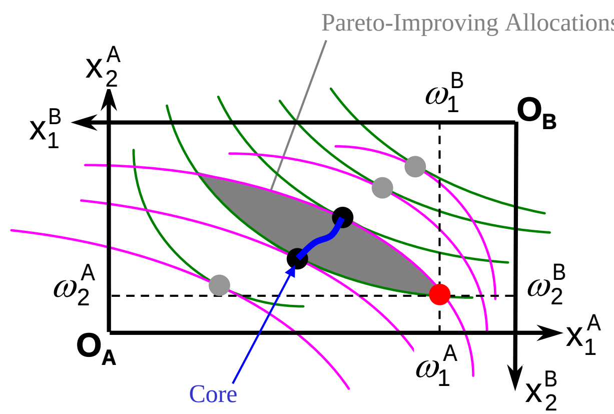
"Contract Curve" is the set of all Pareto-optimal allocations



The Core: Definition

- *Core* - set of all allocations that are:
  - Pareto optimal allocations, and
  - Welfare-improving for both consumers, given original endowments
- Rational trade should achieve allocation in core
- Which core allocation?
  - Depends on allocation mechanism: Competition, Monopoly, negotiation, etc.

"Core"

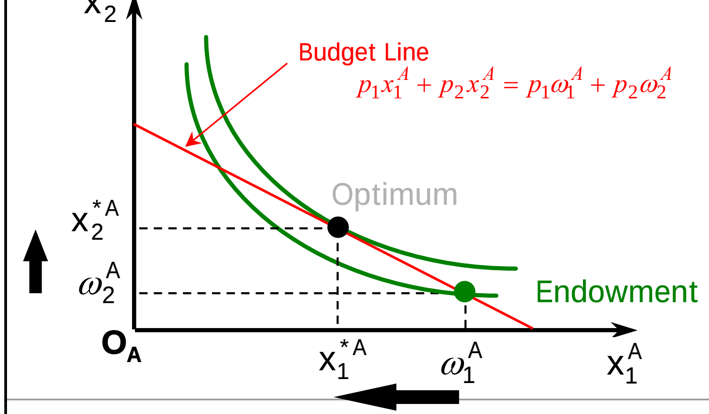


Trade in Competitive Markets

- Suppose final allocation is determined by a competitive price mechanism
  - Consumers are price takers
  - Endowment determines location of budget line (analogous to income constraint)
  - Endowment may not be optimal Consumption bundle
  - Consumers trade to reach optimum
- Note: not realistic in two person case
  - Two groups
  - True for many individuals

Trade in Competitive Markets

For consumer A

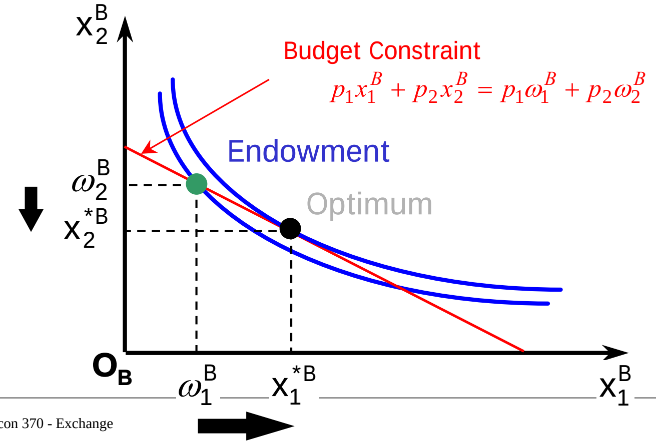


Trade in Competitive Markets

- So, given p_1, p_2, consumer A wants to
  - Reduce consumption of good 1
  - Increase consumption of good 2
  - (Relative to endowments of 1 and 2)
- Thus, *net demands* for commodities 1, 2 are
  - $x_1^{*A} - \omega_1^A$,
  - $x_2^{*A} - \omega_2^A$

Trade in Competitive Markets

For consumer B



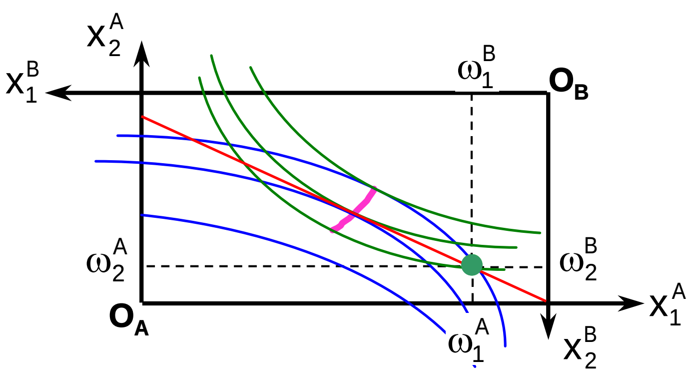
Trade in Competitive Markets

- So, given p_1, p_2, consumer B wants to
  - Increase consumption of good 1
  - Decrease consumption of good 2
  - (Relative to endowments of 1 and 2)
- Thus, *net demands* for commodities 1, 2 are
  - $x_1^{*B} - \omega_1^B$,
  - $x_2^{*B} - \omega_2^B$

General equilibrium

- General Equilibrium occurs for (p_1, p_2) such that both commodity markets clear:
  - $x_1^{*A} + x_1^{*B} = \omega_1^A + \omega_1^B$
  - $x_2^{*A} + x_2^{*B} = \omega_2^A + \omega_2^B$
- Requires (p_1, p_2) tangent to both IC's at same point, so Supply = Demand for both goods
- Recall price line is budget line for both A, B, and endowment point is same for A, B

Trade in Competitive Markets



Trade in Competitive Markets

- At prices (p_1, p_2) both markets clear
- There is a general equilibrium
- Trading in competitive markets achieves a <u>specific</u> Pareto optimal reallocation of endowments
- Pareto optimality in exchange also referred to as *efficiency in consumption*

Efficiency in Consumption

- Efficiency in consumption requires

$$MRS_A = \frac{dx_2^A}{dx_1^A} = \frac{dx_2^B}{dx_1^B} = MRS_B$$

- Or mutually beneficial trades exist
- (MRS's reflect relative valuations of x_1)

Efficiency in Consumption

- Efficiency in consumption achieved with competitive markets
- All consumers face the same p_1 / p_2
- So, all have the same MRS
- So, efficiency in consumption achieved
- Or, Pareto optimal exchange

Walras' Law: Introduction

- Walras' Law is an identity
- Identity
  - a statement that is true for any positive prices (p_1, p_2)
  - these need not be equilibrium prices
- Walras' Law important for constructing general equilibrium models

Walras' Law: Derivation

- Assume every consumer's preferences are well-behaved so, for any (p_1, p_2), spends all of budget
- For consumer A
- $p_1x_2^{*A} + p_2x_2^{*A} = p_1\omega_1^A + p_2\omega_2^A$
- For consumer B
- $p_1x_2^{*B} + p_2x_2^{*B} = p_1\omega_1^B + p_2\omega_2^B$

Walras' Law: Derivation

$$p_1 x_1^{*A} + p_2 x_2^{*A} = p_1 \omega_1^A + p_2 \omega_2^A$$
$$p_1 x_1^{*B} + p_2 x_2^{*B} = p_1 \omega_1^B + p_2 \omega_2^B$$

Summing:

$$p_1 (x_1^{*A} + x_1^{*B}) + p_2 (x_2^{*A} + x_2^{*B}) = p_1 (\omega_1^A + \omega_1^B) + p_2 (\omega_2^B + \omega_2^B)$$

Rearranging:

$$p_1 (x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B) + p_2 (x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B) = 0$$

Walras' Law: Statement

$$p_1 (x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B) + p_2 (x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B) = 0$$

- *Walras' Law*:
  - For any positive prices p_1 and p_2,
  - summed market value of excess demands = zero

First Implication of Walras' Law

Suppose market for commodity A is in equilibrium:

$$x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B = 0$$

Then, Walras' Law implies the other market (for commodity B) *must* be in equilibrium

$$p_1 (x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B) + p_2 (x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B) = 0$$

$$\Rightarrow \mathrm{x}_2^{*\mathrm{A}} + \mathrm{x}_2^{*\mathrm{B}} - \omega_2^{\mathrm{A}} - \omega_2^{\mathrm{B}} = 0$$

General: n – 1 mkts in eq'm, n-th mkt is in equilibrium.

Second Implication of Walras' Law

If there is excess supply of commodity 1:

$$x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B < 0$$

Then, there must be excess demand for 2:

$$p_1 (x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B) + p_2 (x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B) = 0$$

$$\Rightarrow \mathrm{x}_2^{*\mathrm{A}} + \mathrm{x}_2^{*\mathrm{B}} - \omega_2^{\mathrm{A}} - \omega_2^{\mathrm{B}} > 0$$

So, reducing p_1/p_2 moves both mkts to equilibrium

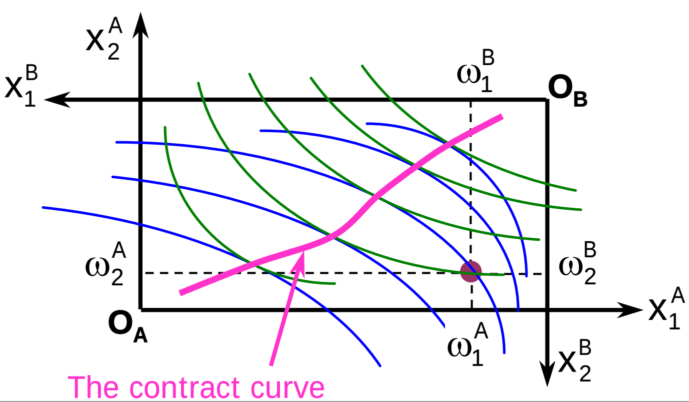
Efficiency In Consumption

- First part of Adam Smith's famous *Invisible Hand Theorem*
  - Efficiency in consumption allocation is an important social goal
  - Private consumers act to maximize own utility
  - But, in competitive markets, private actions achieve social goal, *as if guided by an Invisible Hand*

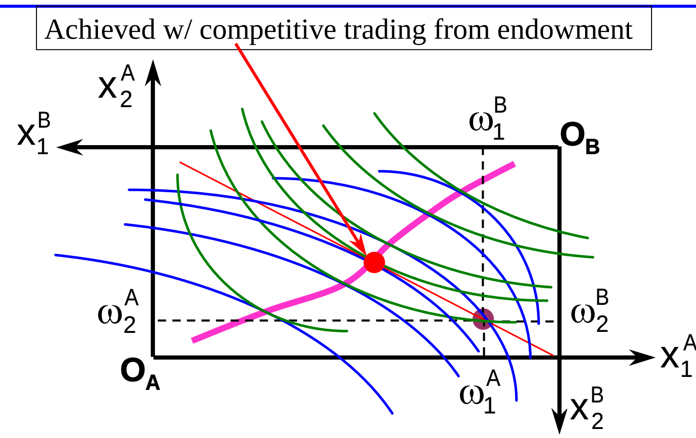
Theorems of Welfare Economics

- 1st Fundamental Theorem:
  - If consumers' preferences are well-behaved (convex, continuous),
  - Then trading in perfectly competitive markets will achieve a Pareto optimal reallocation of the economy's endowment
- 2nd Fundamental Theorem
  - If consumers' preferences are well-behaved (convex, continuous), and
  - Given appropriate rearrangement of endowments among consumers,
  - Any Pareto optimal allocation can be achieved by trading in competitive markets

2nd Fundamental Theorem: Diagram

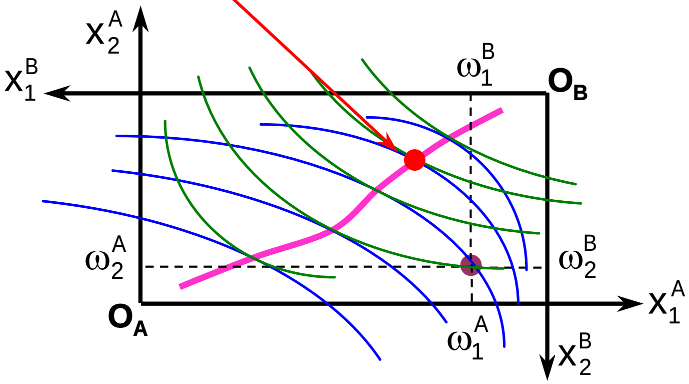


2nd Fundamental Theorem: Diagram



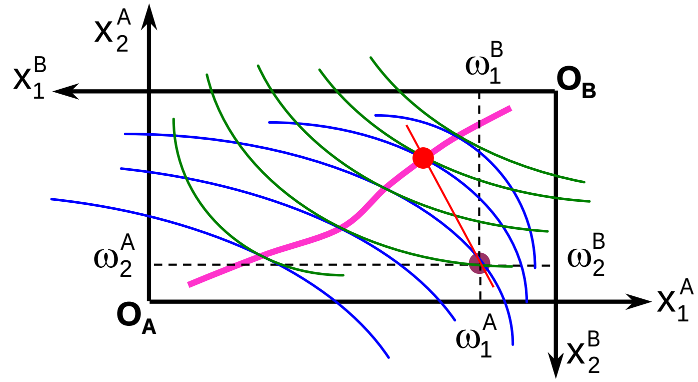
2nd Fundamental Theorem: Diagram

Can this allocation be achieved w/ trading from ω?



2nd Fundamental Theorem: Diagram

No



2nd Fundamental Theorem: Diagram

But can be achieved from alterative endowment θ