Demand and Comparative Statics

ECON 370: Microeconomic Theory

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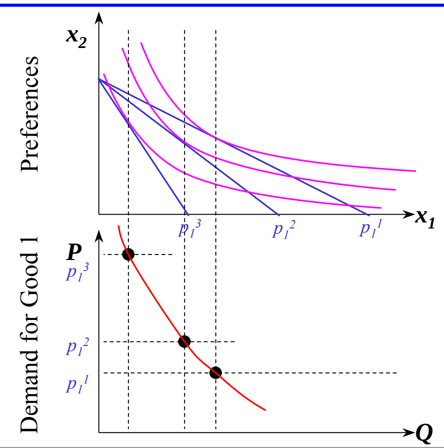
Demand

- Using the tools we have developed up to this point, we can now determine demand for an individual consumer
- We seek demand functions
  - $x_1^* = x_1(p_1, p_2, m)$ and
  - $x_2^* = x_2(p_1, p_2, m)$
- From our previous example
  - Preferences of the form: $u = x^\alpha y^{1-\alpha}$
  - And a budget constraint: $p_x x + p_y y \leq m$
  - Result in demands:
    - $x^* = \alpha m / p_x$
    - $y^* = (1 - \alpha)m / p_y$

Marshallian Demand

- In general, we are interested in tracing out *Marshallian Demand Curves*.
- A *Marshallian Demand Curve* describes how demand for a good changes:
  - As its own price changes, and
  - Holding all other prices and income constant
- Functionally, that means graphing
  - $x_1^* = x_1(p_1, p_2, m)$
  - Versus p_1
  - And holding p_2 and m constant

Graphically



Aggregating Demand

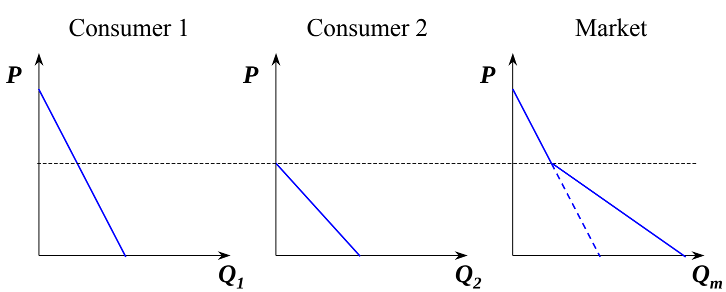
- Ultimately, we want to look at market demand
- That is, we want to see how the demands for many people add together to make up market demand
- If we have *n* people
  - And let $i = 1 \ldots n$
- Two goods, x_1, and x_2 …

Aggregating Demand (cont)

- Consumer *i*'s ordinary demand function for good *j* is
  - $x_j^i(p_1, p_2, m^i)$
  - (notice that we make allowance for different consumers having different income)
- Then aggregate demand for good x_j is:
  - $X_j(p_1, p_2, m^1, \ldots, m^n) = \sum_{i=1}^{n} x_j^i(p_1, p_2, m^i)$
- If all consumers are identical, then
  - $X_j(p_1, p_2, M) = n \times x_j(p_1, p_2, m)$
  - Where $M = n \times m$

Aggregating Demand Graphically

- Graphically, we add demand curves holding price constant
  - That is, horizontally



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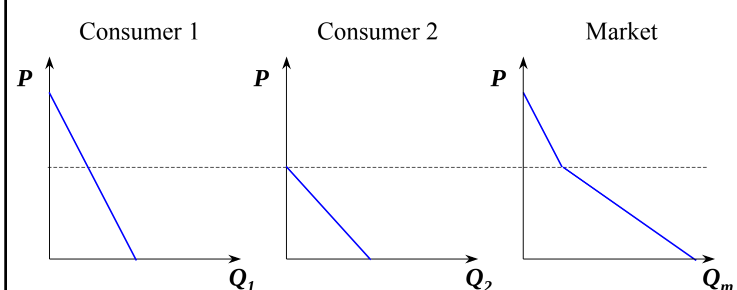


The "Law" of Demand

- What, in Economics is called the "Law" of Demand says
  - Demand curves "always" slope downward
  - That is, Demand for a good "always" decreases as the price for that good increases (holding everything else constant)
- This statement is different from what is in your book—which I will discuss in due time—but it is the more usual statement of the law of demand

The "Law" of Demand (cont)

- The "Law" of Demand is not a law in the sense that it is required by Economic theory
- A good that does not obey the law of demand is theoretically possible and is called a *Giffen* Good
  - (A good that obeys the law of demand is called an *Ordinary* Good)
- However, so far, no one has ever identified a Giffen good
- So the "Law" of demand is a good description of empirical reality

Elasticity of Demand

- One question we are interested in is how responsive demand is to changes is price
- We use something called *Elasticity* to measure it
- In general, Elasticity is the ratio of the percent change of (here) demand to percent change in price
- That is:

$$\varepsilon_i = \frac{\%\Delta x_i}{\%\Delta p_i}$$

Calculating Elasticity

- There are two types of Elasticity:
  - Point Elasticity: $\varepsilon = \dfrac{dx_i / x_i}{dp_i / p_i} = \dfrac{p_i}{x_i} \dfrac{dx_i}{dp_i}$
  - Arc Elasticity: $\varepsilon = \dfrac{\overline{p}_i}{\overline{x}_i} \dfrac{\Delta x_i}{\Delta p_i} = \dfrac{p_i^1 + p_i^2}{x_i^1 + x_i^2} \dfrac{x_i^2 - x_i^1}{p_i^2 - p_i^1}$
- We will almost always use Point Elasticity

Elasticity in General

- There are many different Elasticities
- *Own-Price Elasticity* measures how demand changes as its own price changes
- *Income Elasticity* measures how demand changes as income changes
- *Cross-Price Elasticities* measure how demand changes as the prices of other goods change

Observations on Demand Elasticity

- Own-Price Elasticity may be different at different points on the demand curve
- Since demand generally decreases as price increases, we have $\varepsilon < 0$
  - But usually for convenience we work with $|\varepsilon|$.
- We call demand *Elastic* if $|\varepsilon| > 1$
- We call demand *Inelastic* if $|\varepsilon| < 1$
- In general,
  - when demand is elastic, it responds strongly to changes in price
  - when demand is inelastic, it responds weakly to changes in price

Example: Linear Demand Curve

- Suppose inverse demand is linear
  - $p_i = a - bx_i$
  - $x_i = a/b - p_i / b$
- We calculate $dx_i / dp_i = -1 / b$
- Own-price elasticity of demand is

$$\varepsilon = \frac{p_i}{x_i}\frac{dx_i}{dp_i} = \frac{p_i}{(a-p_i)/b}\left(-\frac{1}{b}\right) = -\frac{p_i}{a-p_i}$$

Example: Points of Interest

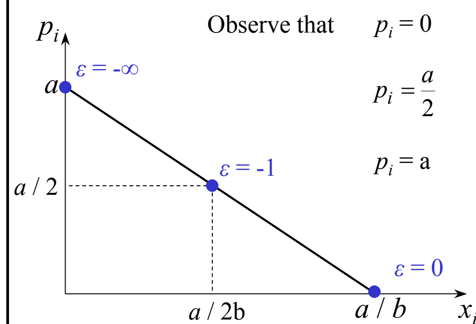
$$p_i = a - bx_i \qquad \varepsilon = -\frac{p_i}{a-p_i}$$

Observe that

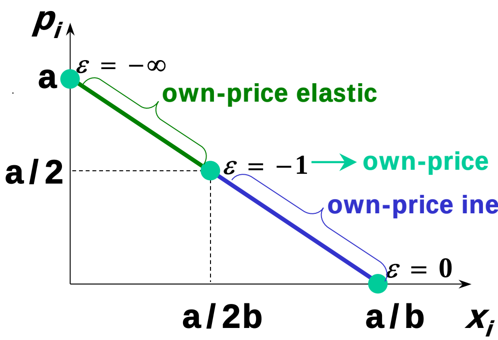
$$p_i = 0 \quad \Rightarrow \quad \varepsilon = -\frac{0}{a-0} = 0$$

$$p_i = \frac{a}{2} \quad \Rightarrow \quad \varepsilon = -\frac{a/2}{a-a/2} = -1$$

$$p_i = \mathrm{a} \quad \Rightarrow \quad \varepsilon = -\frac{a}{a-a} = -\infty$$



Example: Summary



Important Special Cases

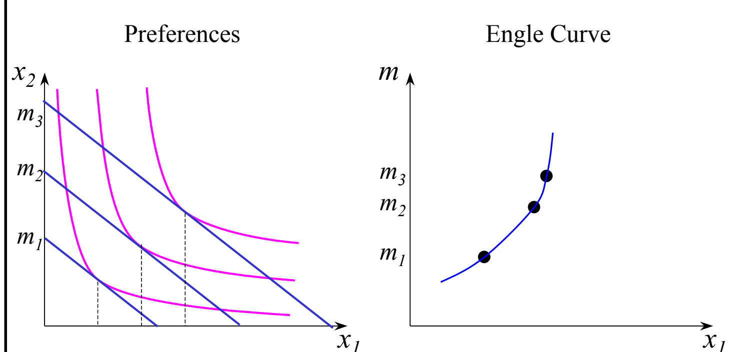
- What is the elasticity of a horizontal demand curve?
- What is the elasticity of a vertical demand curve

Income Elasticity of Demand

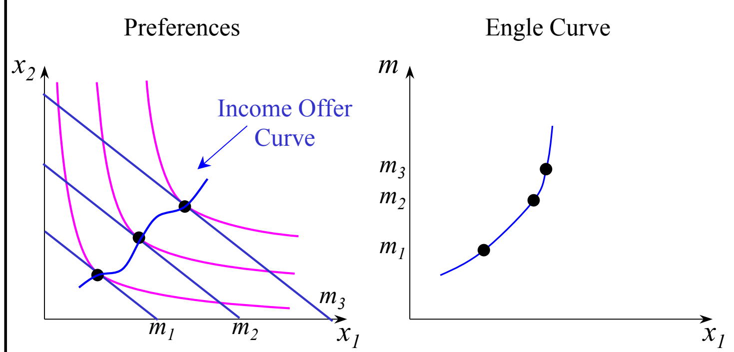
- We are also interested in demand changes as income changes
- We use *Income Elasticity* to measure it, where
  - Income Elasticity for Good i is defined as:

$$\varepsilon_{im} = \frac{m}{x_i}\frac{dx_i}{dm}$$

Income Effects Graphically



Graphical Income Elasticity (cont)



Normal and Inferior Goods

- For most goods demand increases as income increases
  - That is $\varepsilon_{im} > 0$
  - And the Engle curve is sloped forward
  - Such goods are called *Normal* Goods
- However, demand for some goods decreases as income increases
  - That is $\varepsilon_{im} < 0$
  - And the Engle curve is sloped backward
  - Such goods are called *Inferior* Goods
- Note that some goods may be both Normal and Inferior in different price ranges

Inferior to What?

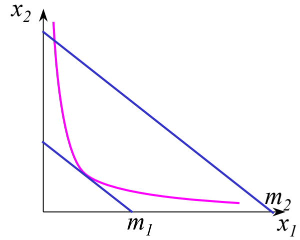
- What exactly is an inferior good, and what is it inferior to?
- Example: the Yugo
  - Very cheap car: Sold for ~ $4,000 in the US in 1986
  - Poor quality and very small
  - Sold moderately well in the US
  - Still in production in Serbia
  - Who do you think bought the cars?
  - What do you think happened when their income increased?
- Inferior to What?
  - To other goods available

Normal, Inferior and Giffen Goods

- A Normal Good cannot be a Giffen Good
- -or- All Giffen Goods are Inferior goods
- (Can you see why?)
- However, most Inferior Goods are <u>not</u> Giffen Goods

Can all goods be inferior?

- All goods cannot be Inferior, that is
- At least one good must be Normal
- Why?



- Are there any preferences that will result in consumption decreasing for both goods when income increases?

Cross-Price Elasticity of Demand

- Finally, we are interested in how demand changes as other prices change
- We use *Cross-Price Elasticity* to measure it, where
  - Cross-Price Elasticity for Good *i* relative to the price of good *j* is defined as:

$$\varepsilon_{ij} = \frac{p_j}{x_i}\frac{dx_i}{dp_j}$$

Substitutes and Complements

- If consumers see two goods as close substitutes for each other, what sign will the cross-price elasticity of demand have?
  - $\varepsilon_{ij} > 0$
- If consumers see two goods as complements for each other, what sign will the cross-price elasticity of demand have?
  - $\varepsilon_{ij} < 0$

Examples

- Perfect Complements/Substitutes
- Homothetic Preferences
- $x = Ap^{\varepsilon}$
  - What makes this interesting?