Consumer Choice

ECON 370: Microeconomic Theory

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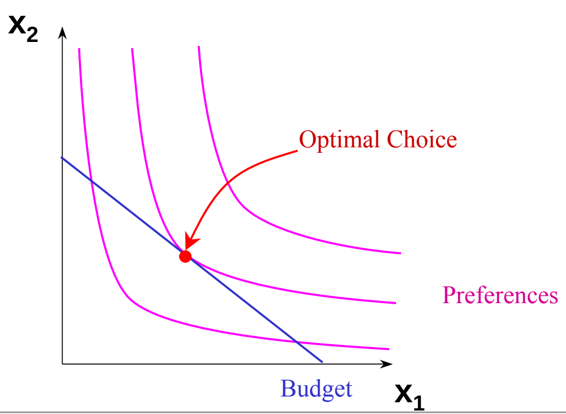
Consumer Equilibrium

- Now we have all the tools we need to analyze consumer choice
- Suppose a consumer faces the constraints
  - $p_1x_1 + p_2x_2 \leq m$ and $x_1 \geq 0, x_2 \geq 0$.
- The consumer's problem is to choose and to *maximize utility* (i.e. to attain highest possible indifference curve) s.t. the above constraints,
- That is, the consumer is choosing the *most preferred* point in the budget set.

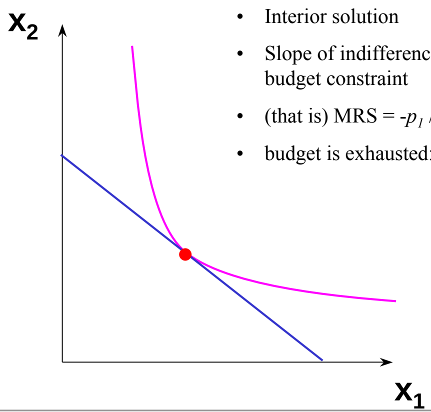
Optimal Choice



Rational Constrained Choice

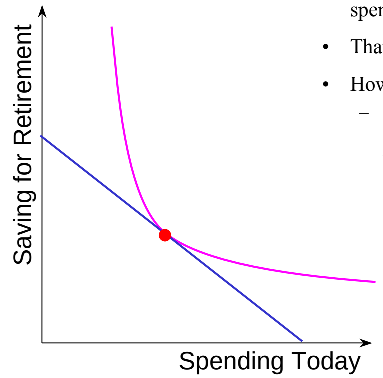
- Provided that this is an "interior" equilibrium,
  - That is, that it involves strictly positive amounts of all goods
- And assuming all our assumptions (A1 – A4) hold:
- Then we have equality of
  - Slope of budget constraint ($-p_1 / p_2$)
  - Slope of indifference curve (MRS)
  - Ratio of marginal utilities (MU_1/MU_2)
- And our budget is exhausted: $p_1x_1^* + p_2x_2^* = m$
- Interpretation
  - Willingness to trade (-slope of IC) = p_1 / p_2
  - Marginal utility per dollar equalized across goods

Characteristics of Optimal Choice



- Interior solution
- Slope of indifference curve = Slope of budget constraint
- (that is) MRS = $-p_1 / p_2$
- budget is exhausted: $p_1x_1^* + p_2x_2^* = m$

What about savings?



- We said that generally all money is spent
- That is $p_1x_1^* + p_2x_2^* = m$
- How do we deal with savings?
  - By assuming our consumption choices include spending today v. spending in the future.

Finding the Optimum

- We will demonstrate two methods for finding the optimal choice
  - MRS = p_1 / p_2
  - Maximizing the Utility function
- (At least) one other that I will not cover:

$$\frac{MU_1}{p_1} = \frac{MU_2}{p_2}$$

  - Mathematically, it is indistinguishable from the first

Example

- Assume for this example that
  - Preferences are Cobb-Douglass
  - $U = x^{\alpha}y^{1-\alpha}$
  - Budget constraint is $p_xx + p_yy \leq m$

MRS Solution

In order to calculate MRS, we first calculate Marginal Utilities

$$MU_x = \frac{\partial U}{\partial x} = \alpha x^{\alpha-1} y^{1-\alpha} \qquad MU_y = \frac{\partial U}{\partial y} = (1-\alpha) x^{\alpha} y^{-\alpha}$$

Then we calculate MRS as:

$$MRS = \frac{dy}{dx} = \frac{\partial u / \partial x}{\partial u / \partial y} = \frac{MU_x}{MU_y} = \frac{\alpha x^{\alpha-1} y^{1-\alpha}}{(1-\alpha) x^{\alpha} y^{-\alpha}} = \boxed{\frac{\alpha}{1-\alpha}\frac{y}{x}}$$

Since MRS equals the slope of the budget line, we get:

$$\frac{\alpha}{1-\alpha}\frac{y}{x} = \frac{p_x}{p_y}$$

MRS Solution (cont)

We will use x^* and y^* to represent our solution

$$\frac{\alpha}{1-\alpha}\frac{y^*}{x^*} = \frac{p_x}{p_y} \quad \text{(from previous page)}$$

Solving for y^*:

$$y^* = \frac{1-\alpha}{\alpha}\frac{p_x}{p_y} x^*$$

Since all money is spent, we also have: $p_x x^* + p_y y^* = m$

So: $$p_x x^* + p_y \frac{1-\alpha}{\alpha}\frac{p_x}{p_y} x^* = p_x x^* + \frac{1-\alpha}{\alpha} p_x x^* = \frac{p_x x^*}{\alpha} = m$$

MRS Solution (cont 2)

Finally:

If $\frac{p_x x^*}{\alpha} = m$ Then $x^* = \frac{\alpha m}{p_x}$

Substituting this into $y^* = \frac{1-\alpha}{\alpha}\frac{p_x}{p_y} x^*$ (from before)

Gives $y^* = \frac{(1-\alpha)m}{p_y}$

So, our solution is: $x^* = \frac{\alpha m}{p_x} \qquad y^* = \frac{(1-\alpha)m}{p_y}$

Maximization Solution

- Our problem is:
  - $\max\{u(x, y)\} = \max\{x^{\alpha} y^{1-\alpha}\}$
  - Subject to the requirement that $p_x x + p_y y \leq m$.
- Remembering that all money will be spent, we have:
  - $p_x x + p_y y = m$
- From here, the simplest way to proceed is to substitute the modified constraint into problem to be solved:
  - $y = (m - p_x x) / p_y$
- Substituting into the original problem gives:

$$\max\left\{x^{\alpha} y^{1-\alpha}\right\} = \max\left\{x^{\alpha} (m - p_x x)^{1-\alpha} p_y^{\alpha-1}\right\}$$

Maximization (cont)

Remembering our calculus, maximizing a function requires:

$$\frac{df(x)}{dx}=0 \qquad \text{and} \qquad \frac{d^2 f(x)}{dx^2}\le 0$$

So $\quad \max\left\{x^{\alpha}(m-p_x x)^{1-\alpha}\, p_y^{\alpha-1}\right\}$

$$\Rightarrow \frac{d}{dx}x^{\alpha}(m-p_x x)^{1-\alpha}\, p_y^{\alpha-1}=0$$

$$\Rightarrow p_y^{\alpha-1}\left[\alpha x^{\alpha-1}(m-p_x x)^{1-\alpha}-p_x(1-\alpha)x^{\alpha}(m-p_x x)^{-\alpha}\right]=0$$

Maximization (cont 2)

Since all this mess equals zero, we can eliminate common terms:

$$\cancel{p_y^{\alpha-1}}\left[\alpha x^{\cancel{\alpha}-1}(m-p_x x)^{1-\cancel{\alpha}}-p_x(1-\alpha)\cancel{x^{\alpha}}\cancel{(m-p_x x)^{-\alpha}}\right]=0$$

$$\Rightarrow \alpha\left(\frac{m-p_x x}{x}\right)-(1-\alpha)p_x=0$$

$$\Rightarrow \alpha(m-p_x x)-(1-\alpha)p_x x=\alpha m-\alpha p_x x-p_x x+\alpha p_x x=\alpha m-p_x x=0$$

$$\Rightarrow x=\frac{\alpha m}{p_x}$$ Exactly as before

Maximization (cont 3)

Verifying the Second Order Condition:

$$\frac{d}{dx}p_y^{\alpha-1}\left[\alpha x^{\alpha-1}(m-p_x x)^{1-\alpha}-p_x(1-\alpha)x^{\alpha}(m-p_x x)^{-\alpha}\right]$$

(reorganizing it a bit)

$$=p_y^{\alpha-1}\left[\alpha\frac{d}{dx}\left(\frac{m}{x}-p_x\right)^{1-\alpha}-p_x(1-\alpha)\frac{d}{dx}\left(\frac{m}{x}-p_x\right)^{-\alpha}\right]$$

$$=p_y^{\alpha-1}\left[-\alpha(1-\alpha)\frac{m}{x^2}\left(\frac{m}{x}-p_x\right)^{-\alpha}-p_x\alpha(1-\alpha)\frac{m}{x^2}\left(\frac{m}{x}-p_x\right)^{-\alpha-1}\right]<0$$

Cobb-Douglas Observation

- Note that for Cobb-Douglas utility function
  - Demands are linear in income
  - Expenditure shares are constant
  - Expenditure shares sum to one

$$x^*=\frac{\alpha m}{p_x} \qquad\qquad y^*=\frac{(1-\alpha)m}{p_y}$$

"Corner" Solutions

- An "interior" solution was earlier described as:
  - involving strictly positive amounts of all goods
  - And all our assumptions (A1 – A4) hold
- If a solution does not meet these requirements, it is called a "Corner" Solution
- Other methods have to be used to solve it

Corner Example