Optimal Behavior

ECON 370: Microeconomic Theory

Summer 2004 – Rice University

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Short Run v. Long Run

- Short Run
  - Some inputs cannot be varied by the firm
  - Free-entry into the market is limited
- Long Run
  - All inputs can be varied
  - Free-Entry into the market is unrestricted
- Example
  - Boeing and Machine Tools

Initial Assumptions

- Firms maximize profits
  - Remember: maximizing profits means MR = MC
- Competitive Firm
  - The firm treats all input and output prices as fixed
- Short Run
  - In particular, the firm can only vary one input

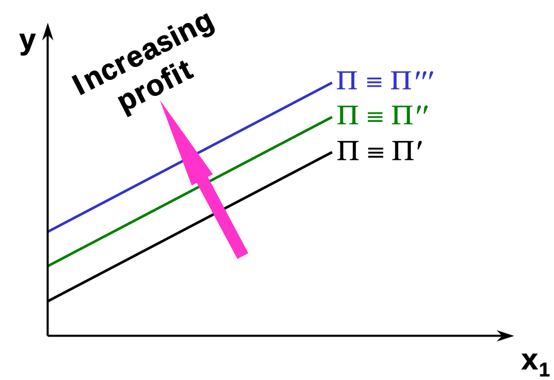
Variables

- The SR Profit Max Problem
  - Choose production plan
  - to maximize profits
  - given production function
- Let p_y = price of output
- Let p_i = price of input i
- Let x_i = Amount of input i supplied
- Let $y = F(x_1, x_2)$ = output
- Assume amount of input 2 is fixed at $\tilde{x}_2$

Short-Run Iso-Profit Lines: Introduction

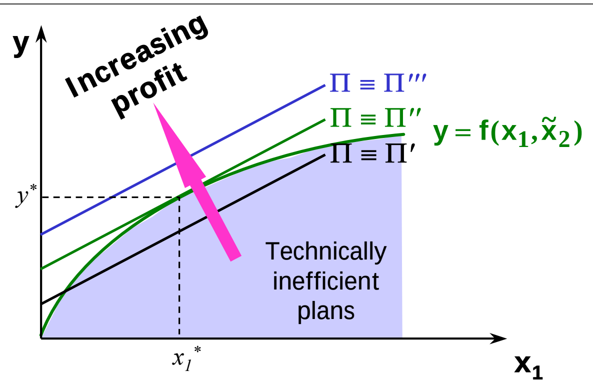
- Iso-profit line P is
  - All production plans providing profit level P
  - Whether feasible or not
- Equation is
  - $\Pi \equiv p_y y - p_1 x_1 - p_2 \tilde{x}_2$ or $y = \frac{p_1}{p_y} x_1 + \frac{\Pi + p_2 \tilde{x}_2}{p_y}$
  - so slope = $\frac{p_1}{p_y}$
  - and intercept = $\frac{(\Pi + p_2 \tilde{x}_2)}{p_y}$

Short-Run Iso-Profit Lines: Graph



Short-Run Profit-Maximization: Graph

SR production function, technology set for $x_2 \equiv \tilde{x}_2$



Mathematical Analysis

- Revenue R = $p_y y = p_y F(x_1, \tilde{x}_2)$
- Costs C = $p_1 x_1 + p_2 \tilde{x}_2$
- Since optimal production requires MR = MC:

$$\frac{d}{dx_1} R = \frac{d}{dx_1} C$$

$$\Rightarrow \frac{d}{dy} p_y y \frac{dy}{dx_1} = \frac{d}{dx_1}(p_1 x_1 + p_2 x_2)$$

$$\Rightarrow p_y \frac{d}{dx_1} F(x_1, \tilde{x}_2) = p_1 \qquad \Rightarrow \boxed{p_y MP_1 = p_1}$$

example

Comparative Statics

- What happens if p_y increases/decreases
- What happens if p_1 increases/decreases

Long Run

- In the long run, all factors can be changed
- It turns out, in the long run, the marginal product rule applies to all inputs
- That is
  - For all *I*
  - $p_y MP_i = p_i$

example

Cost Minimization

- This is a different way of getting at profit maximization
- This approach asks
  - Given a proposed output level
  - What is the most efficient (least-cost) input mix
- Cost minimization is required for profit maximization to occur
  - Why?
- Final result is a cost function $c(y, p_1, p_2)$
  - Which we usually abbreviate to $c(y)$

Problem Statement

- The Cost Minimization Problem is:

$$\min_{x_1, x_2} p_1 x_1 + p_2 x_2$$

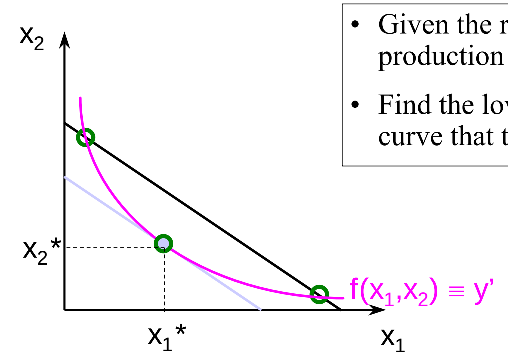
$$\text{Subject to} \quad F(x_1, x_2) = y$$

- Solution $x_1^*(p_1,p_2,y)$ and $x_2^*(p_1,p_2,y)$ are firm's conditional demands for inputs 1, 2
- Smallest possible total cost for producing y is: $c(p_1,p_2,y) = p_1 x_1^*(p_1,p_2,y) + p_2 x_2^*(p_1,p_2,y)$

Iso-Cost Lines

- Iso-Cost Line contains all input bundles that cost same amount
- E.g., given p_1 and p_2, the iso-cost line for cost \$*c* has equation
  - $p_1 x_1 + p_2 x_2 = c$
  - Or x2 = $(c / p_2) - (p_1 / p_2) x_1$
- Slope is $-(p_1 / p_2)$
- Intercept is (c / p_2)

The Cost-Minimization Problem: Graph



- Given the relevant production isoquant
- Find the lowest iso-cost curve that touches it

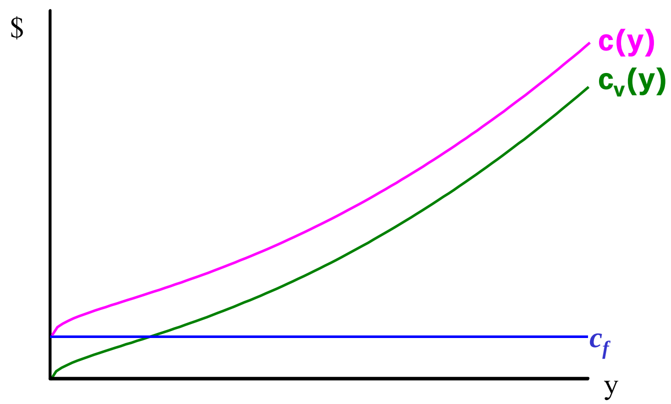
Mathematically

- Note from the previous graph…
- that for interior solutions…
- The slope of the iso-cost curve = the slope of the production isoquant at the optimal point
- That is
  - TRS = (p_1 / p_2)
- Example: Cobb-Douglas

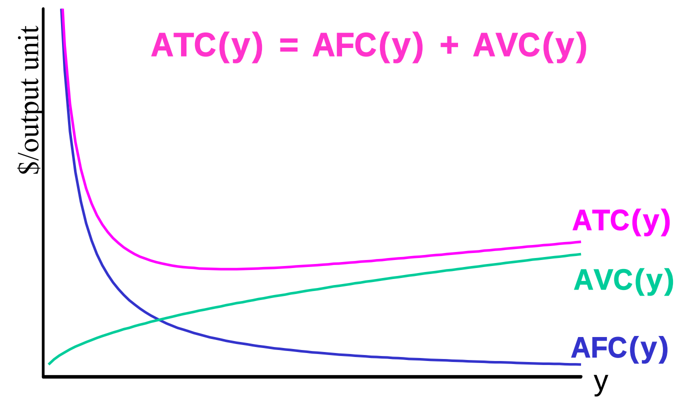
Taxonomy of Costs

- We assume that the cost function can be broken into the form
- $c(y) = c_v(y) + c_f$
- Where
  - $c_v(y)$ are variable costs
  - c_f are fixed costs
- We define
  - Average (total) Costs = A(T)C = $c(y) / y$
  - Average Variable Costs = AVC = $c_v(y) / y$
  - Average Fixed Costs = AFC = c_f / y

Total Costs: Graph



Average Total Cost Curve: Graph



Marginal and Variable Cost Functions

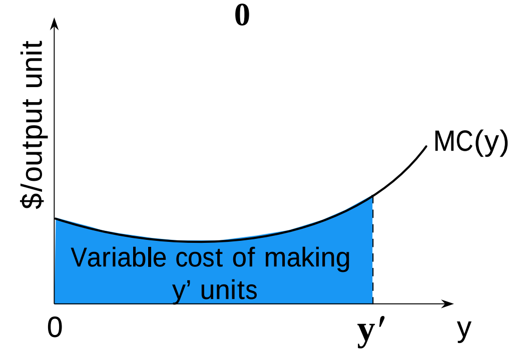
- Since MC(y) is derivative of $c_v(y)$, then $c_v(y)$ must be integral of MC(y)

$$MC(y) = \frac{\partial c_v(y)}{\partial y}$$

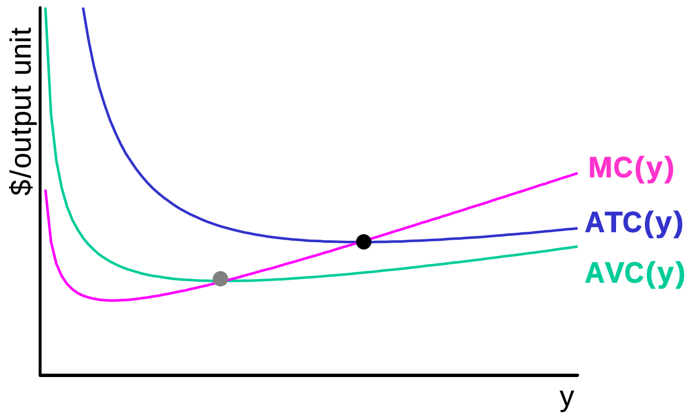
$$\Rightarrow c_v(y) = \int_0^y MC(z)dz$$

Marginal and Variable Cost Functions

$$c_v(y') = \int_0^{y'} MC(z)dz$$



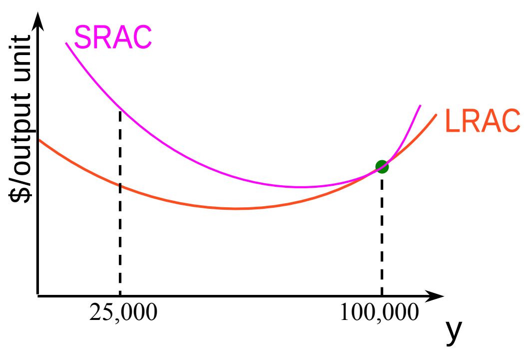
MC, AC Function Intersections: Graph



Long Run v. Short Run

- All costs can be adjusted in the Long Run
- Long run Average Total Cost cannot be higher than short run Average Total Cost
  - Why
- Example
  - In Long Run, Demand of 100,000 units per year anticipated
  - Built a manufacturing facility for sized for 100,000 units
  - In short run, there is a recession, and one year only 25,000 units demanded

SR and LR Marginal Cost Curves: Graph



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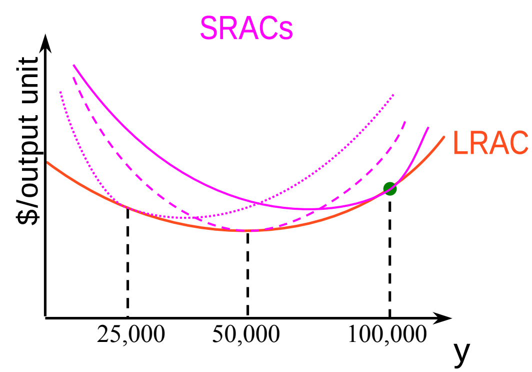
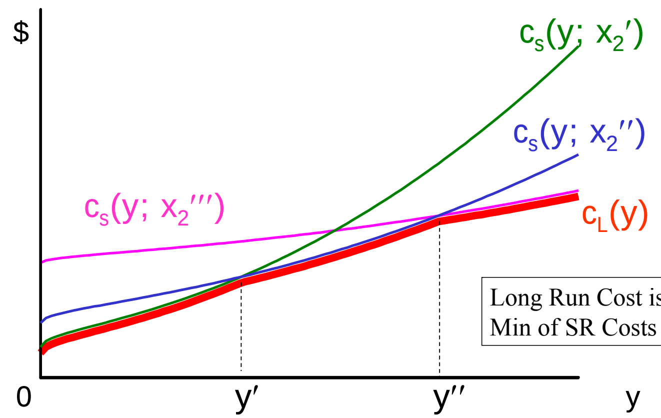
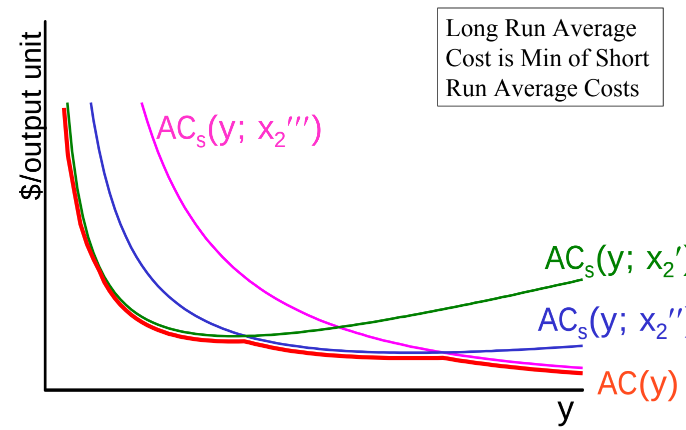


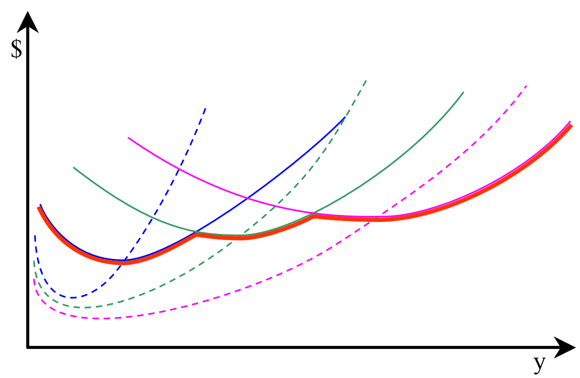
SR and LR Total Cost Curves



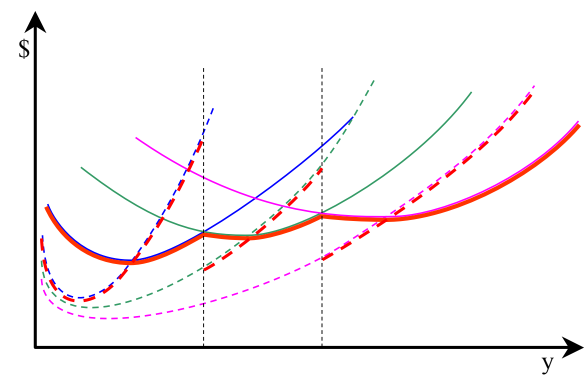
SR and LR Average Total Cost Curves



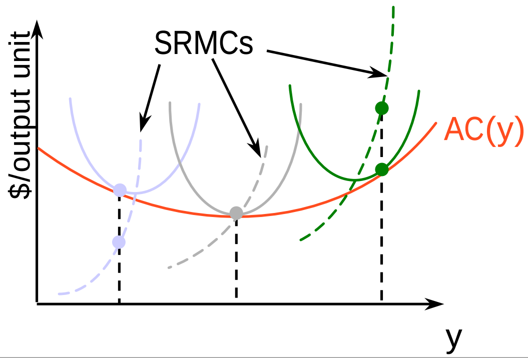
SR & LR Marginal Cost Curves



SR & LR Marginal Cost Curves



SR and LR Marginal Cost Curves: Graph



SR and LR Marginal Cost Curves: Graph

For each y>0,
LR-MC=SR-MC
for SR chosen by firm