

Economics 370

Microeconomic Theory

Problem Set 4

Due **30 June 2004**. Please hand it in at the end of the **30 June 2004** lecture.

- 1) For the production function $f(x_1, x_2) = 2\sqrt{x_1} + x_2^2$:
 - a) show whether the marginal product of factor 1 (respectively, factor 2) increases, stays constant or decreases as the amount of that factor alone is varied.
 - b) show that it does not satisfy the definition of increasing returns to scale, nor constant returns to scale nor decreasing returns to scale. [Hint: find a combination of inputs such that doubling the amount of both inputs will more than double the amount of output, and find another combination of inputs such that doubling the amount of both inputs will less than double the output.]
- 2) The Private Automobile Corporation has two plants for producing cars. Plant A produces according to $f_A(x_1, x_2) = \min\{x_1, 2x_2\}$ and plant B produces according to $f_B(x_1, x_2) = \min\{2x_1, x_2\}$, where x_1 and x_2 are the inputs.
 - a) Graph and label the isoquant for 40 cars at plant A. On the same diagram graph and label the isoquant for 40 cars at plant B.
 - b) Suppose that the firm wishes to produce 20 cars at each plant. How much of each input will the firm need to produce 20 cars at plant A, and how much of each input will the firm need to produce 20 cars at plant B. Label with an a on your diagram, the point representing the total amount of each of the two inputs that the firm needs to produce a total of 40 cars, 20 at each plant.
 - c) Label with a b on your graph the point that shows how much of each of the inputs is needed altogether if the firm is to produce 10 cars at plant A and 30 cars at plant B. Label with a c on your graph the point that shows how much of each of the two inputs is needed in total if it is to produce 30 cars at plant A and 10 cars at plant B. Draw the firm's isoquant for producing 40 units of output (i.e. 40 cars) if it can split production in any manner between the two plants. Is the technology available to this firm convex?
- 3) A firm has two factors and a production function $f(x_1, x_2) = x_1^{1/2} x_2^{1/4}$. The price of the output is 4. Factor 1 receives a wage of w_1 and factor 2 receives a wage of w_2 . Solve for the (long-run) profit-maximizing choice of inputs (x_1^*, x_2^*) . How much output will it produce? How much profit will it make?
- 4) A firm uses labor and machines to produce output according to the production function $f(x_1, x_2) = 4\sqrt{x_1 x_2}$, where x_1 is the quantity of labor employed and x_2 is the quantity of machines used. The cost of labor is \$40 per unit and the cost of using a machine is \$10.
 - a) Draw an isocost line for this firm, showing combinations of labor and machines that cost \$400 and another isocost line showing combinations that cost \$200. What is the slope of these isocost lines?

- b) Suppose that the firm wants to produce its output in the cheapest possible way. Find the number of machines it would use per worker.
 - c) On the same graph you drew your two isocost lines, sketch the production isoquant corresponding to an output of 40. Calculate the amount of labor and the quantity of machines that are used to produce 40 units of output in the cheapest possible way, given the above factor prices. Calculate the cost of producing 40 units at these factor prices.
 - d) What amount of labor and what amount of machines would the firm use to produce y units of output in the cheapest possible way? How much would this cost?
- 5) Mary Magnolia wants to open a flower shop, the Petal Pusher, in a new mall. She has her choice of three different floor sizes, 200 square feet, 500 square feet, or 1,000 square feet. The monthly rent will be \$1 a square foot. Mary estimates that if she has F square feet of floor space and sells y bouquets a month, her variable costs will be $c_v(y) = y^2 / F$ per month.
- a) If she has 200 square feet of floor space, write down her marginal cost function and her average cost function. At what amount of output is average cost minimized?
 - b) If she has 500 square feet, write down her marginal cost function and her average cost function. At what amount is average cost minimized?
 - c) If she has 1,000 square feet, write down her marginal cost function and her average cost function. At what amount is average cost minimized?
 - d) Illustrate in a diagram, Mary's (short-run) average cost curve and her (short-run) marginal cost curve if she has 200, 500 or 1,000 square feet of floor space. On the same diagram draw Mary's long-run average cost and long-run marginal cost curves.